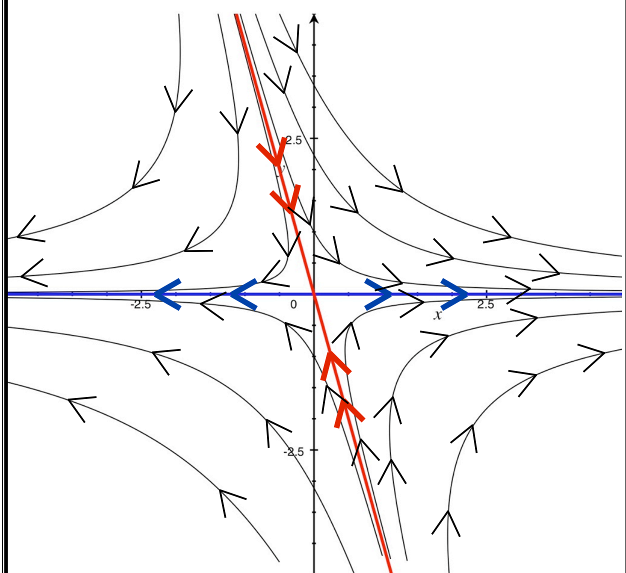
Day 16: July 22nd

- **Chapter: 3.6 Second Order Linear Equations**
  - Homework:
    - Page 336 #1,3, 7, 9, 13, 15, 17, 21, 23.
- **Chapter: 3.7 Trace-Determinant Plane**
  - Homework:
    - Page 352 #2, 3, 5, 9, 11, 14.
- **Chapter: 4.1 Forced Harmonic Oscillators**
- **LAB 2 posted due Wed. July 28th**

Review: Real Distinct Eigenvalues

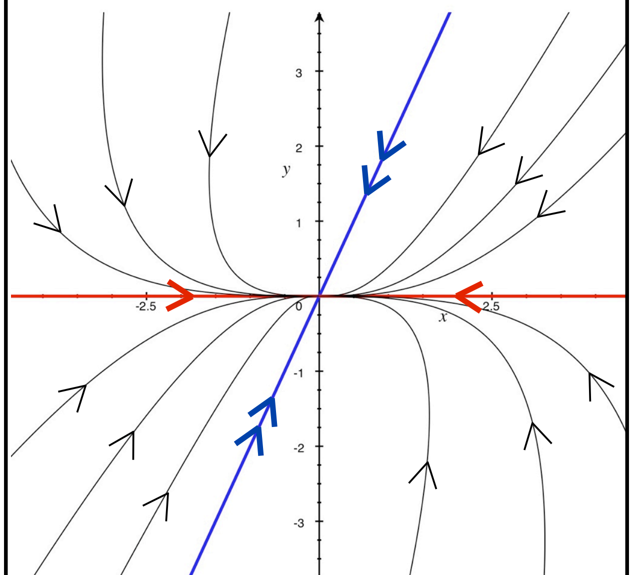
Saddle

$$\lambda_1 < 0 < \lambda_2$$



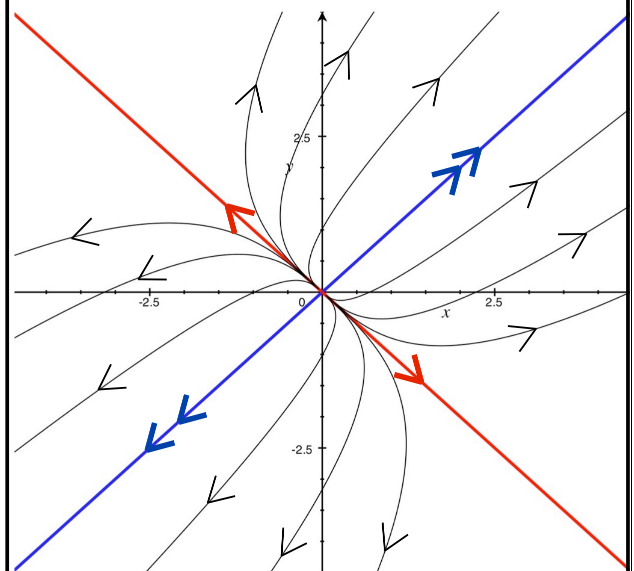
Sink

$$\lambda_2 < \lambda_1 < 0$$



Source

$$0 < \lambda_1 < \lambda_2$$



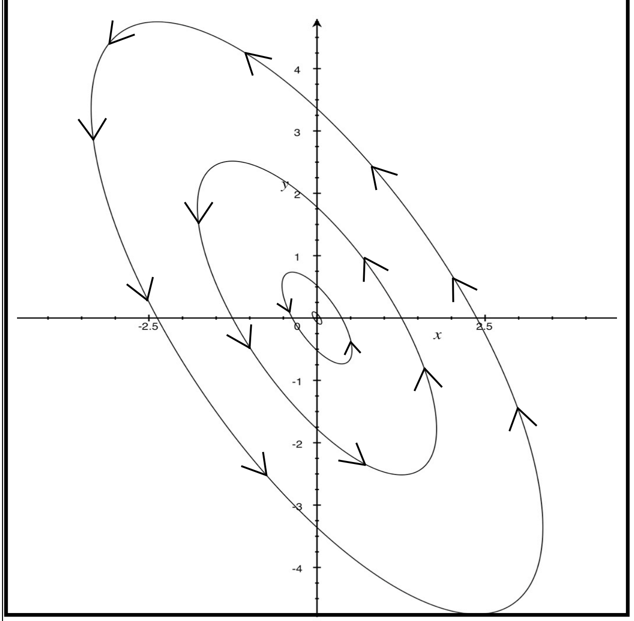
$$\lambda_1, \lambda_2 \in \mathbb{R}$$

Review: Complex Eigenvalues

$$\lambda = a \pm \mathrm{i}b$$

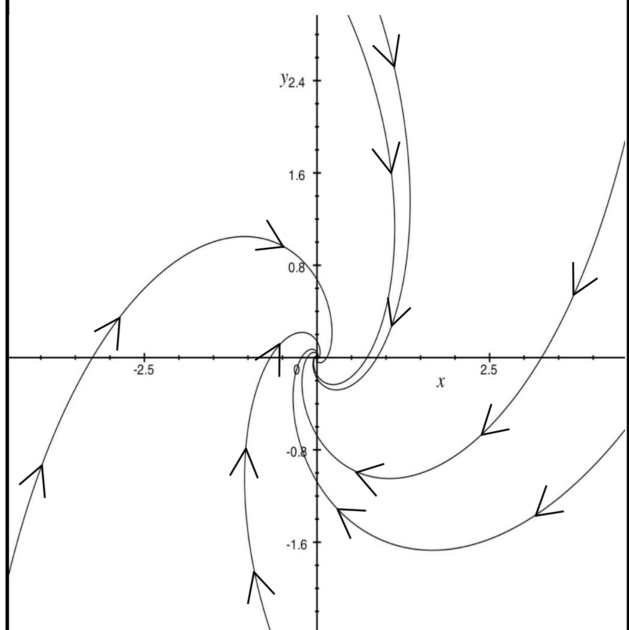
Center

$$a = 0$$



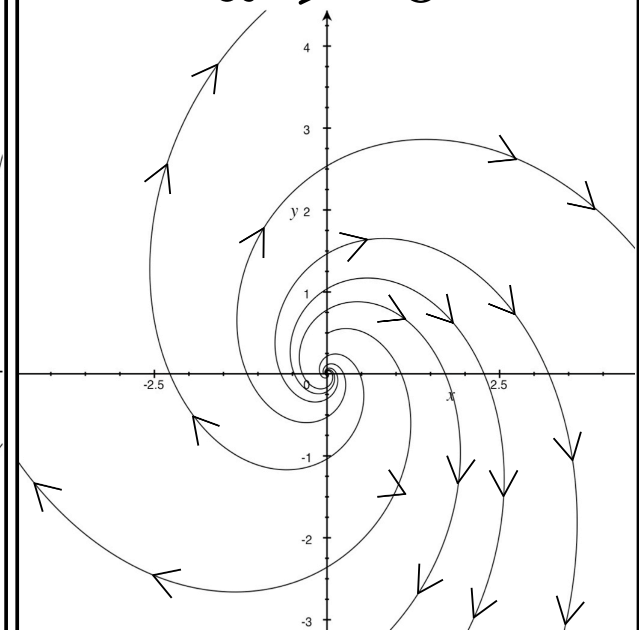
Spiral Sink

$$a < 0$$



Spiral Source

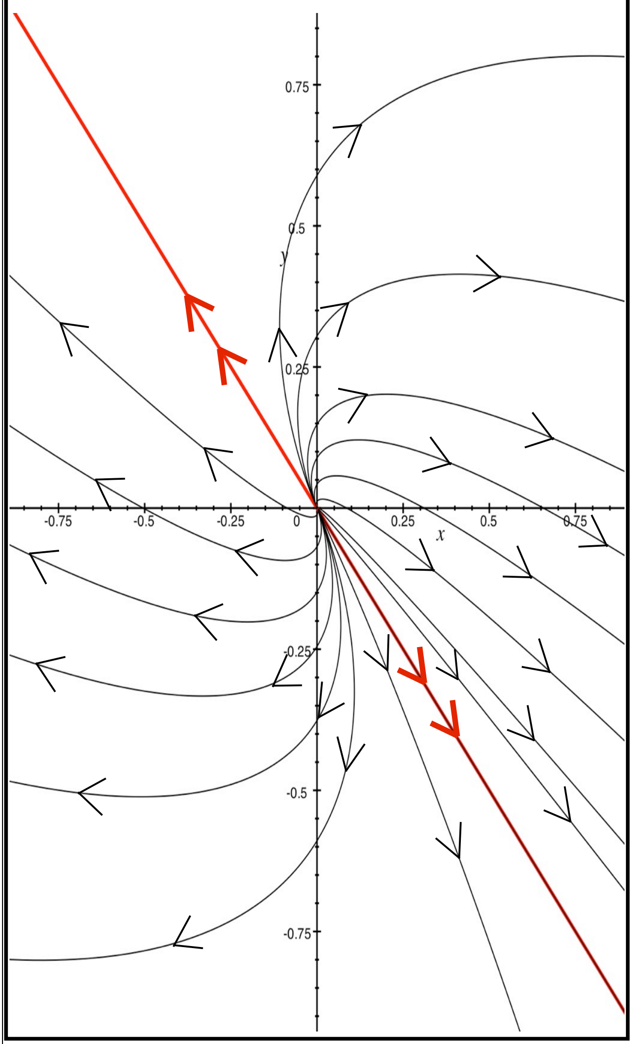
$$a > 0$$



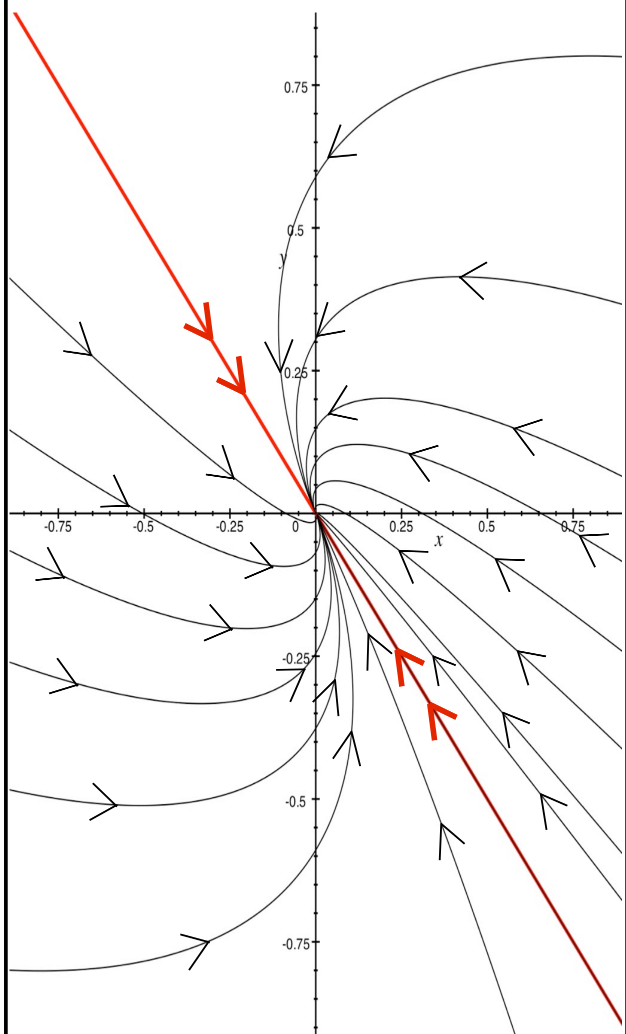
In all cases rotation can be clockwise or counter clockwise.

Review: Real Repeated Eigenvalues

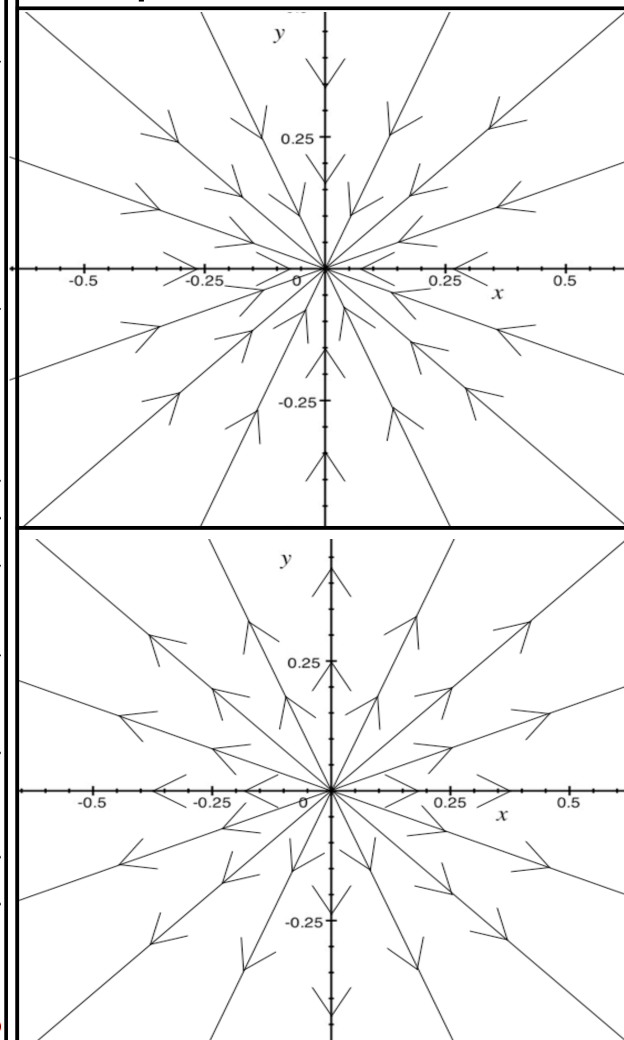
$\lambda > 0$



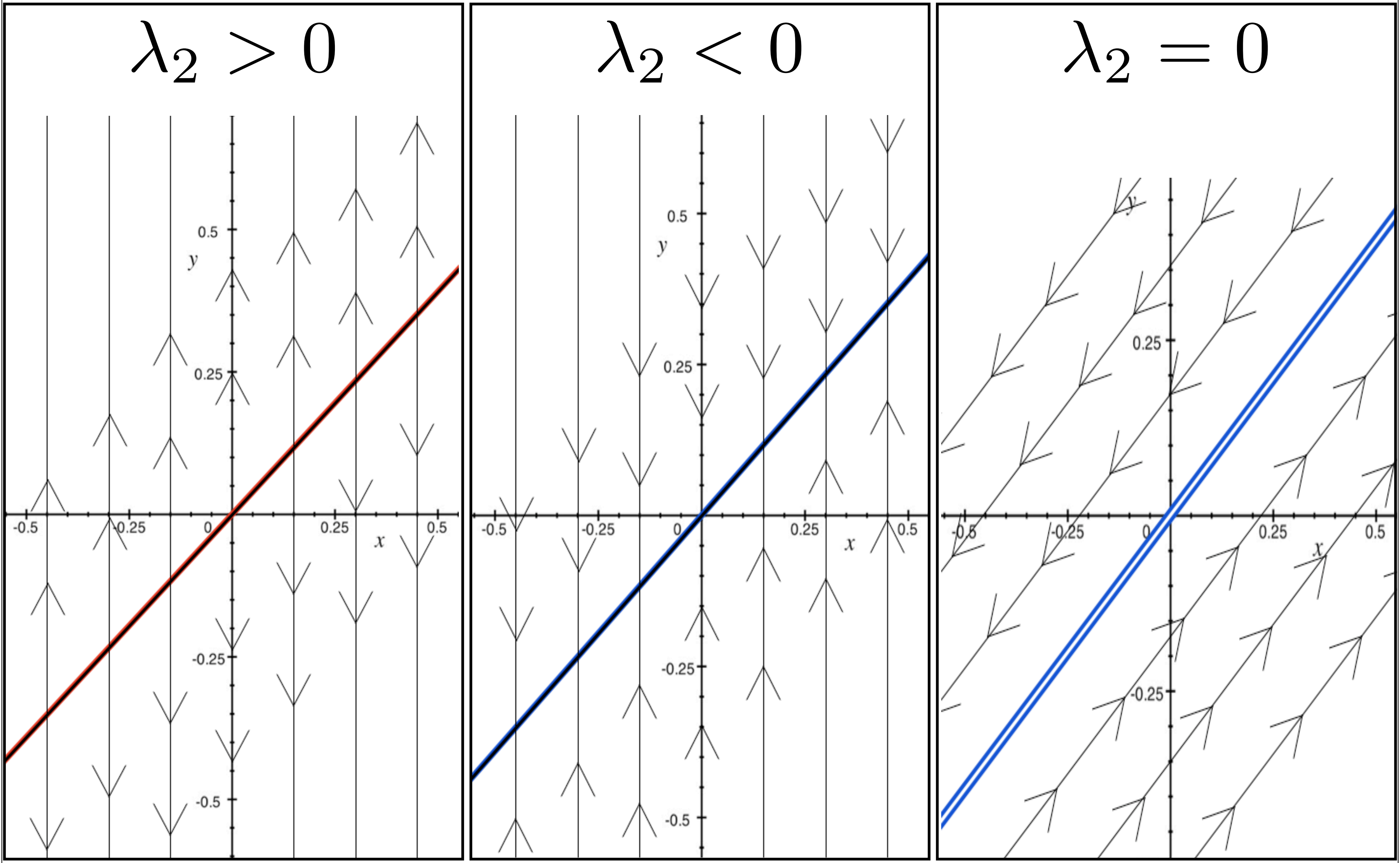
$\lambda < 0$



special case



Review: Zero Eigenvalue



Trace-Determinant Pane

- Solving a system: $\vec{\mathbf{Y}}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \vec{\mathbf{Y}}$

$$\det \begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = 0$$

$$\lambda^2 - (\underbrace{a+d}_{\substack{\uparrow \\ \mathrm{tr}(\mathbf{A})}})\lambda + (\underbrace{ad-bc}_{\substack{\uparrow \\ \det(\mathbf{A})}}) = 0$$

$$\lambda^2 - \mathrm{tr}(\mathbf{A})\lambda + \det(\mathbf{A}) = 0$$

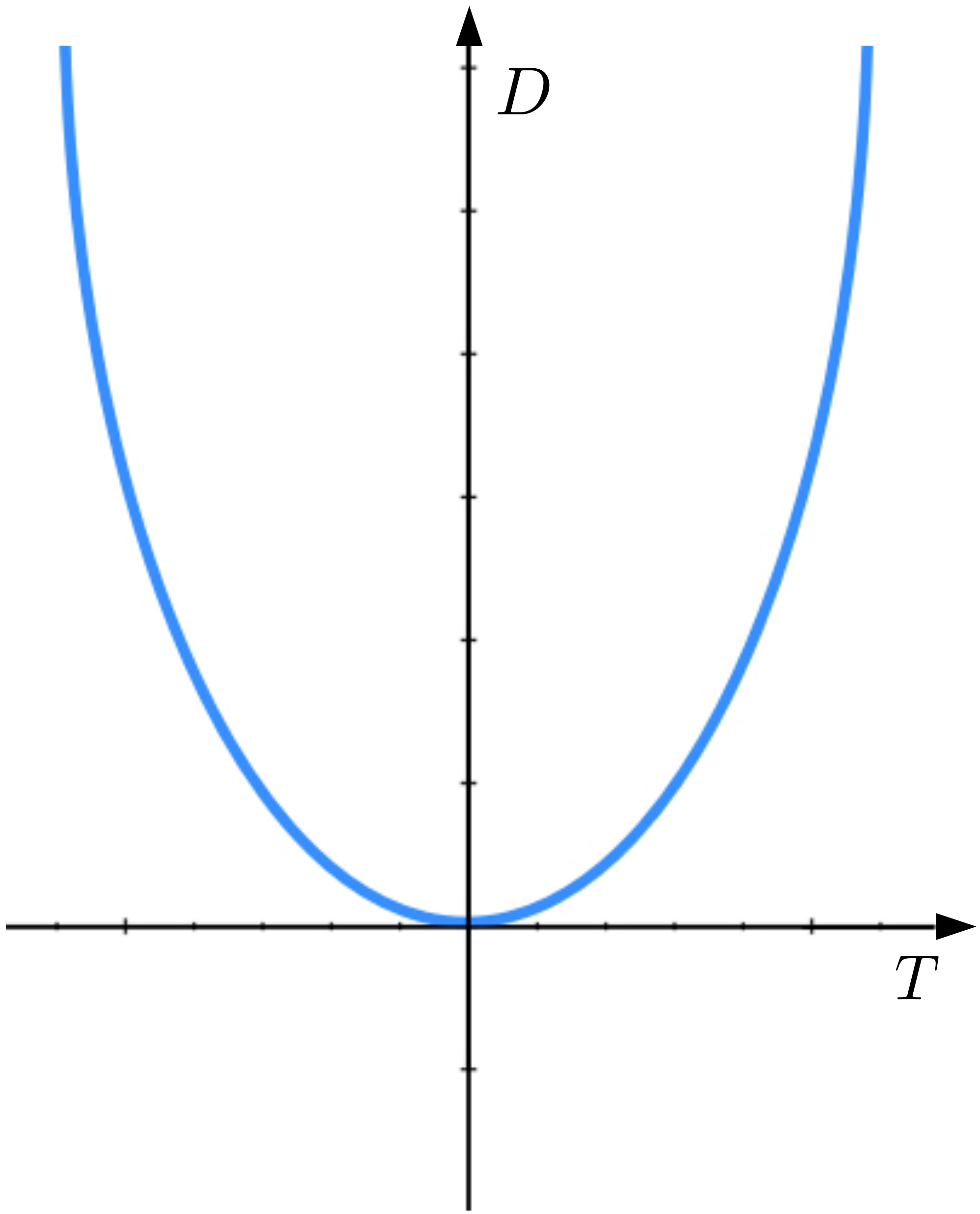
Trace-Determinant Pane

$$\lambda^2 - T\lambda + D = 0$$

- roots: $\dfrac{T \pm \sqrt{T^2 - 4D}}{2}$

<u>eigenvalues</u>

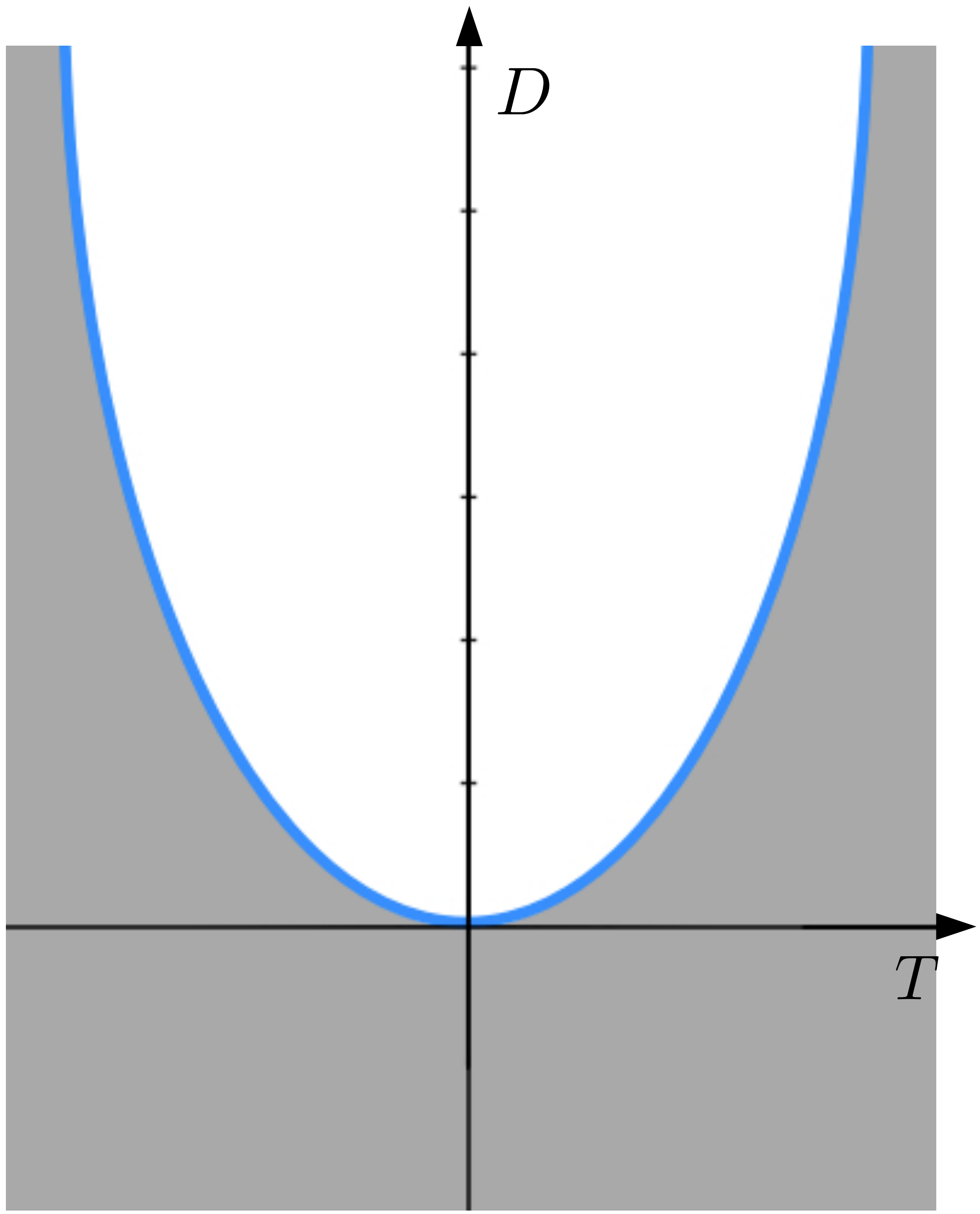
- $T^2 - 4D > 0$ real distinct
- $T^2 - 4D < 0$ complex
- $T^2 - 4D = 0$ repeated



$$T^2 - 4D = 0$$

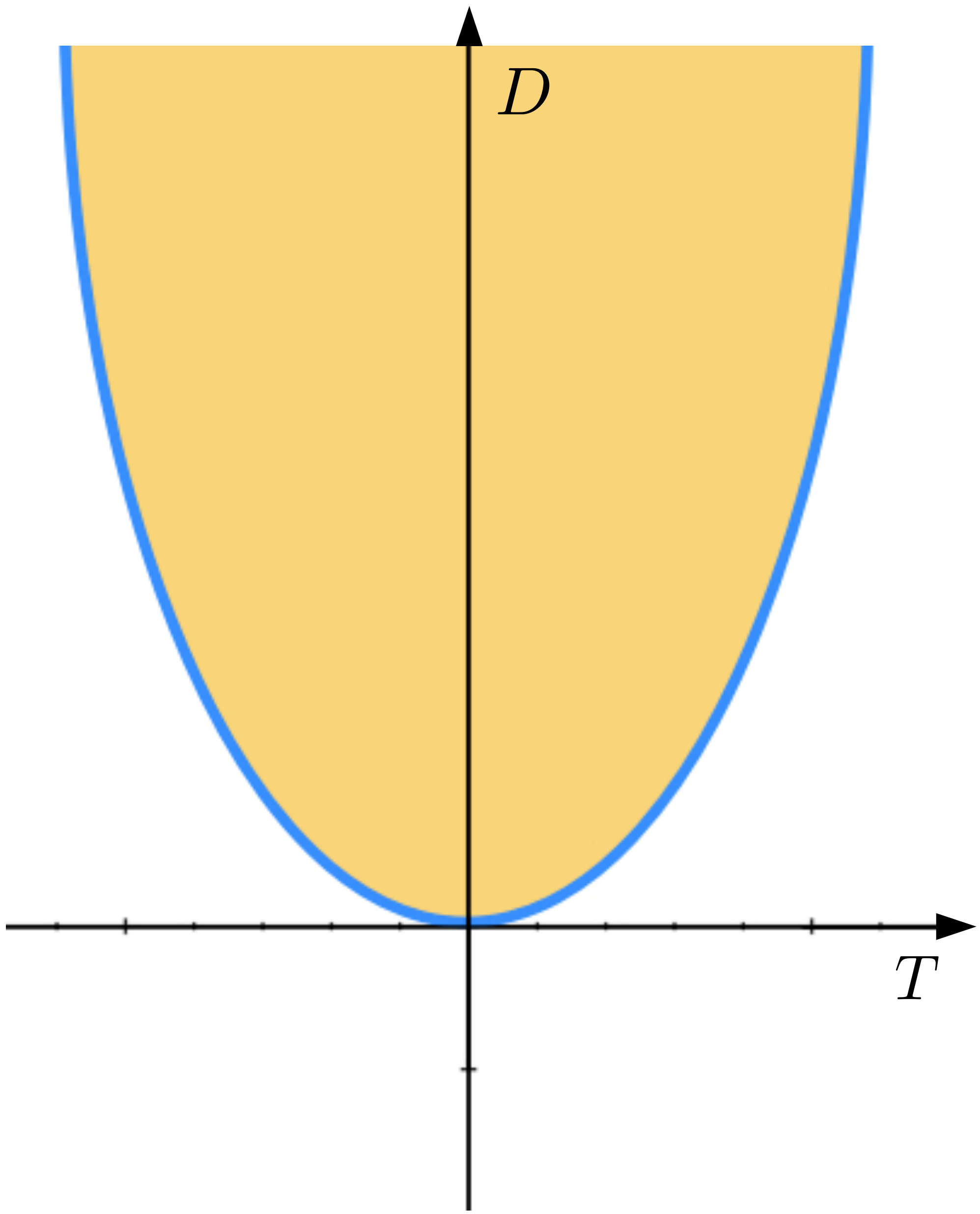
$$D = \frac{T^2}{4}$$

repeated eigenvalue



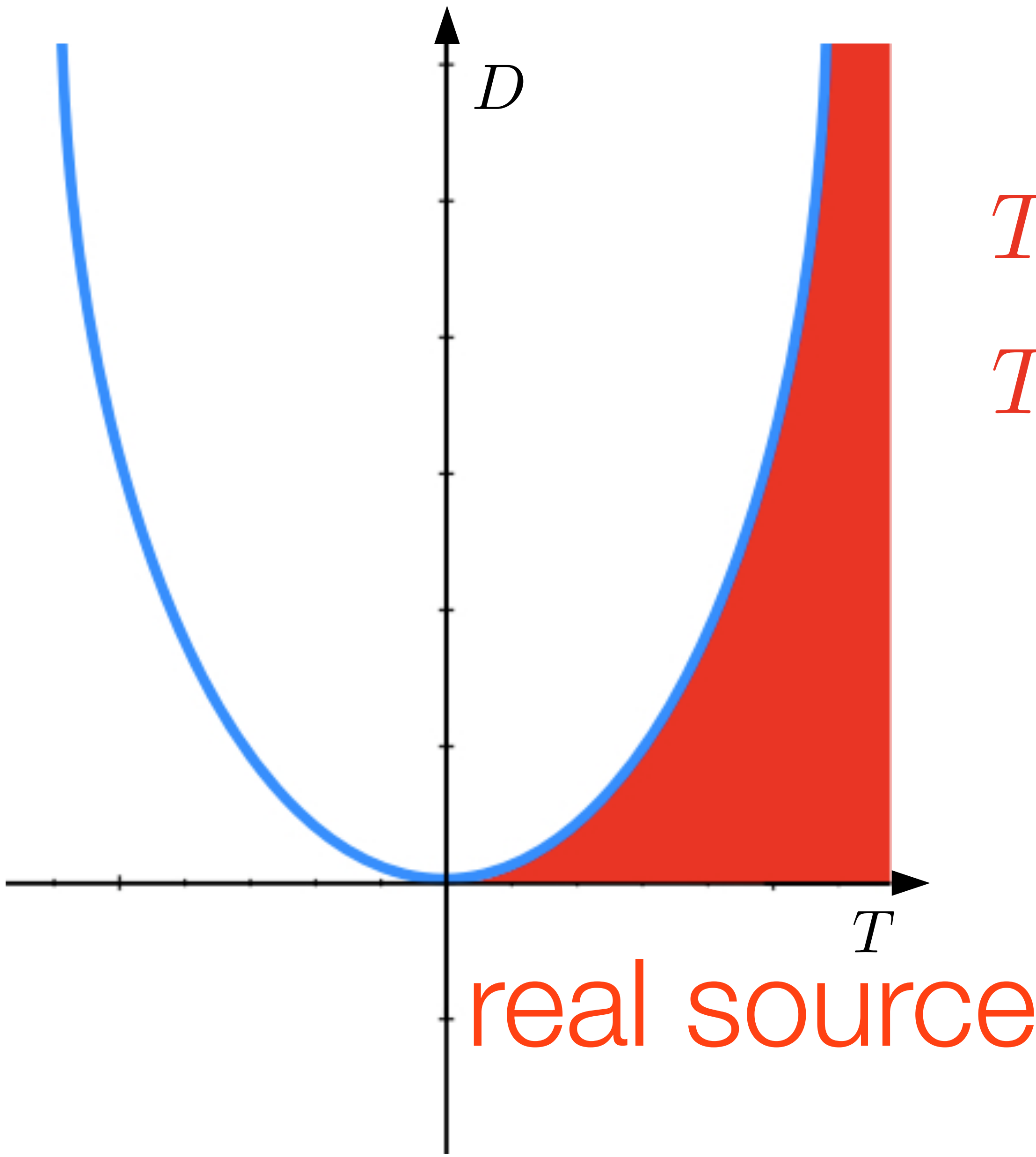
$T^2 - 4D > 0$

real eigen-value



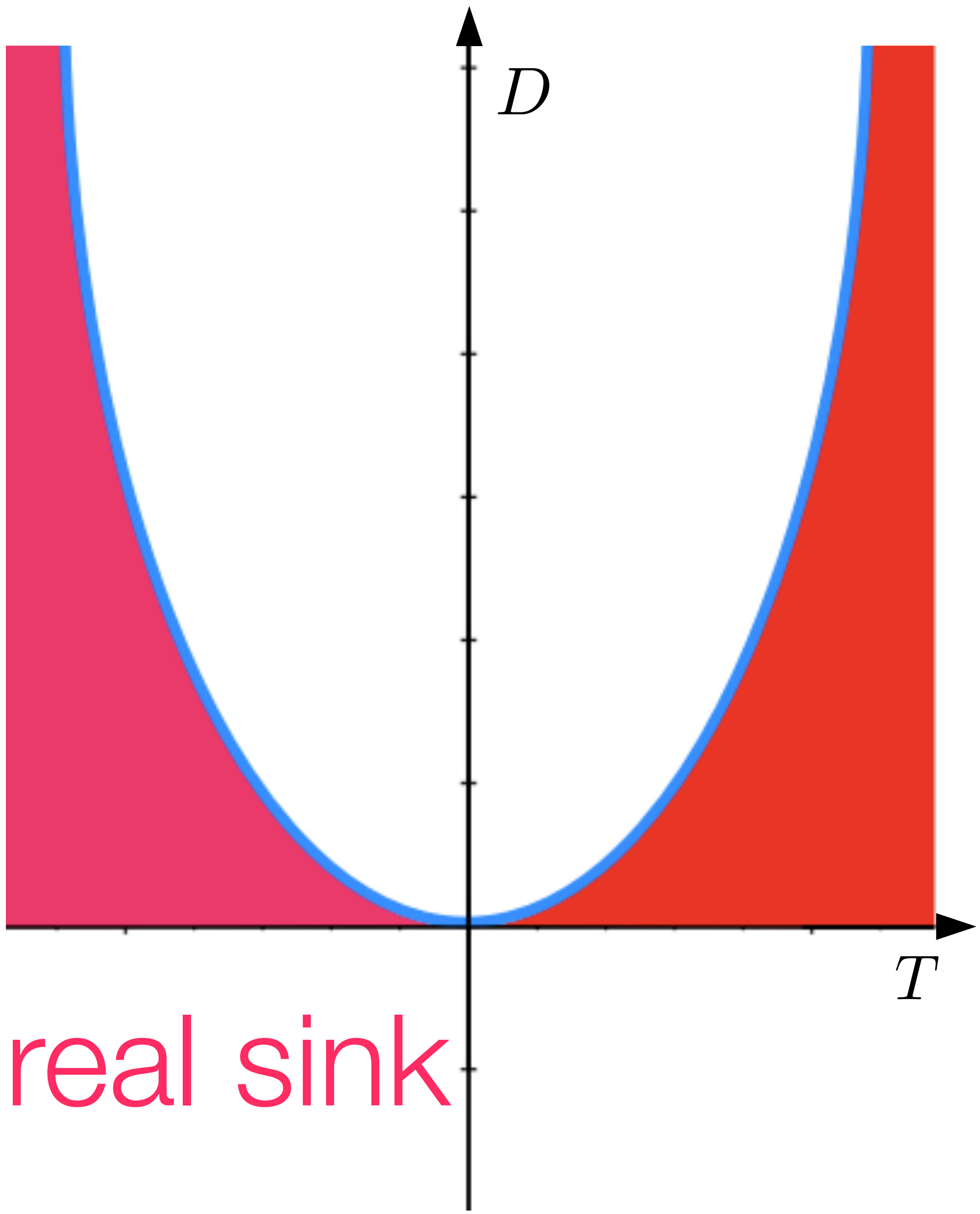
$$T^2 - 4D < 0$$

imaginary eigenvalue



$$T^2 - 4D > 0$$

$$T > 0$$



$$T^2 - 4D > 0$$

$$T < 0$$

real sink

- When: $D < 0$
    - one eigenvalue positive other negative.
- Proof:

roots: $$\frac{T \pm \sqrt{T^2 - 4D}}{2}$$

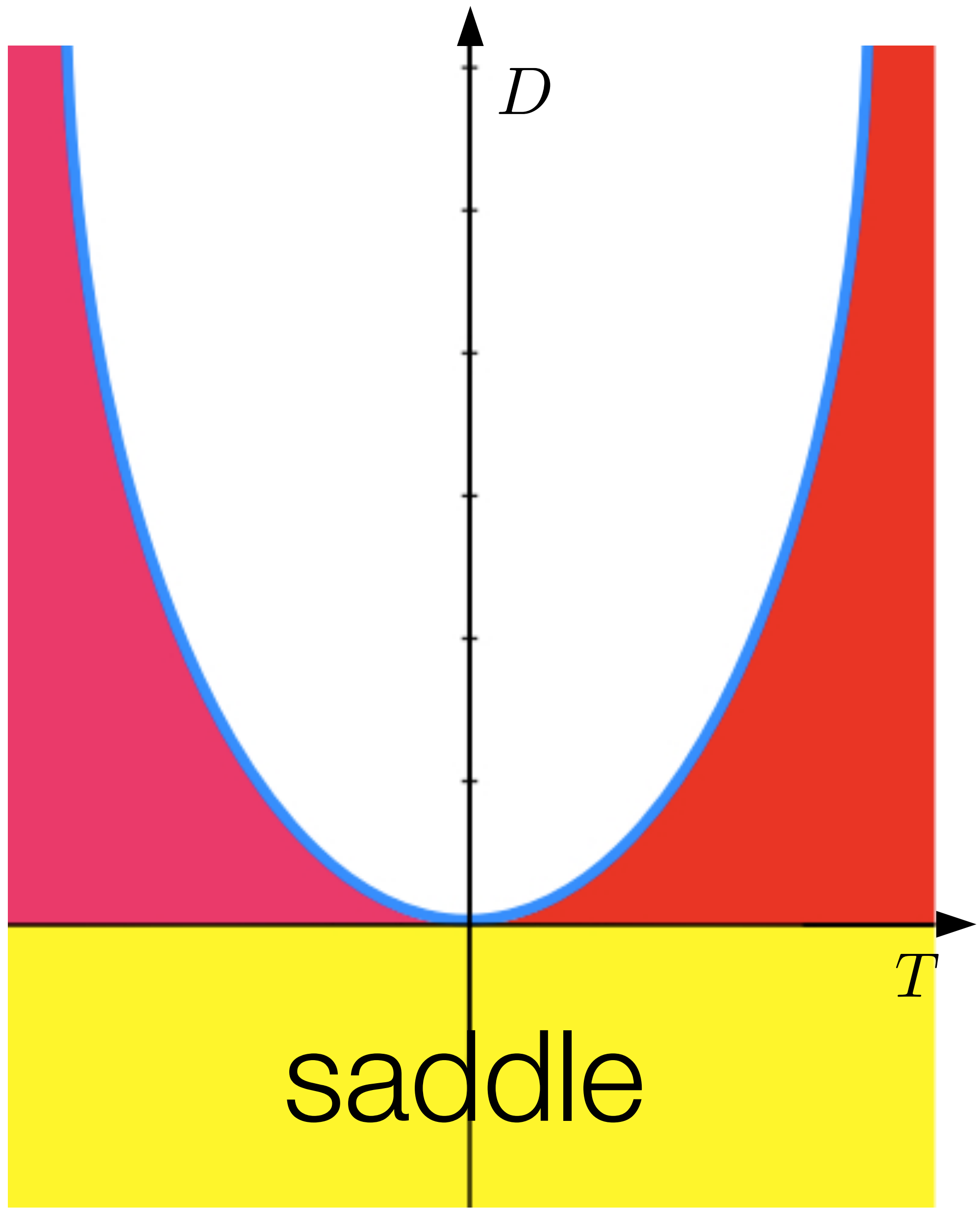
since $D < 0$ $\quad T^2 - 4D > T^2$

- one positive: $+\sqrt{T^2 - 4D} > |T| \geq T$

$$0 > T - \sqrt{T^2 - 4D}$$

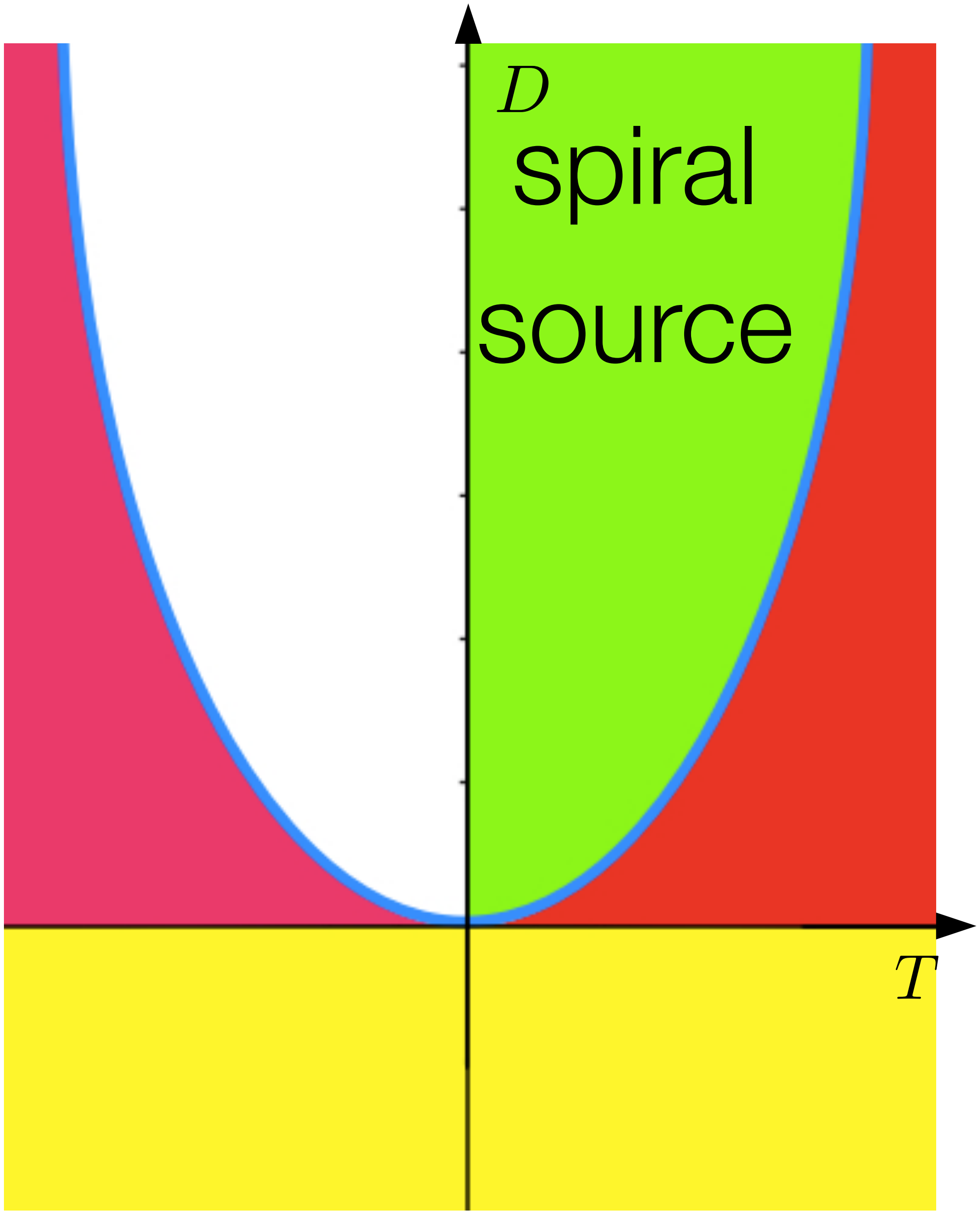
- one negative: $-\sqrt{T^2 - 4D} < -|T| \leq T$

saddle $\quad 0 < T + \sqrt{T^2 - 4D}$



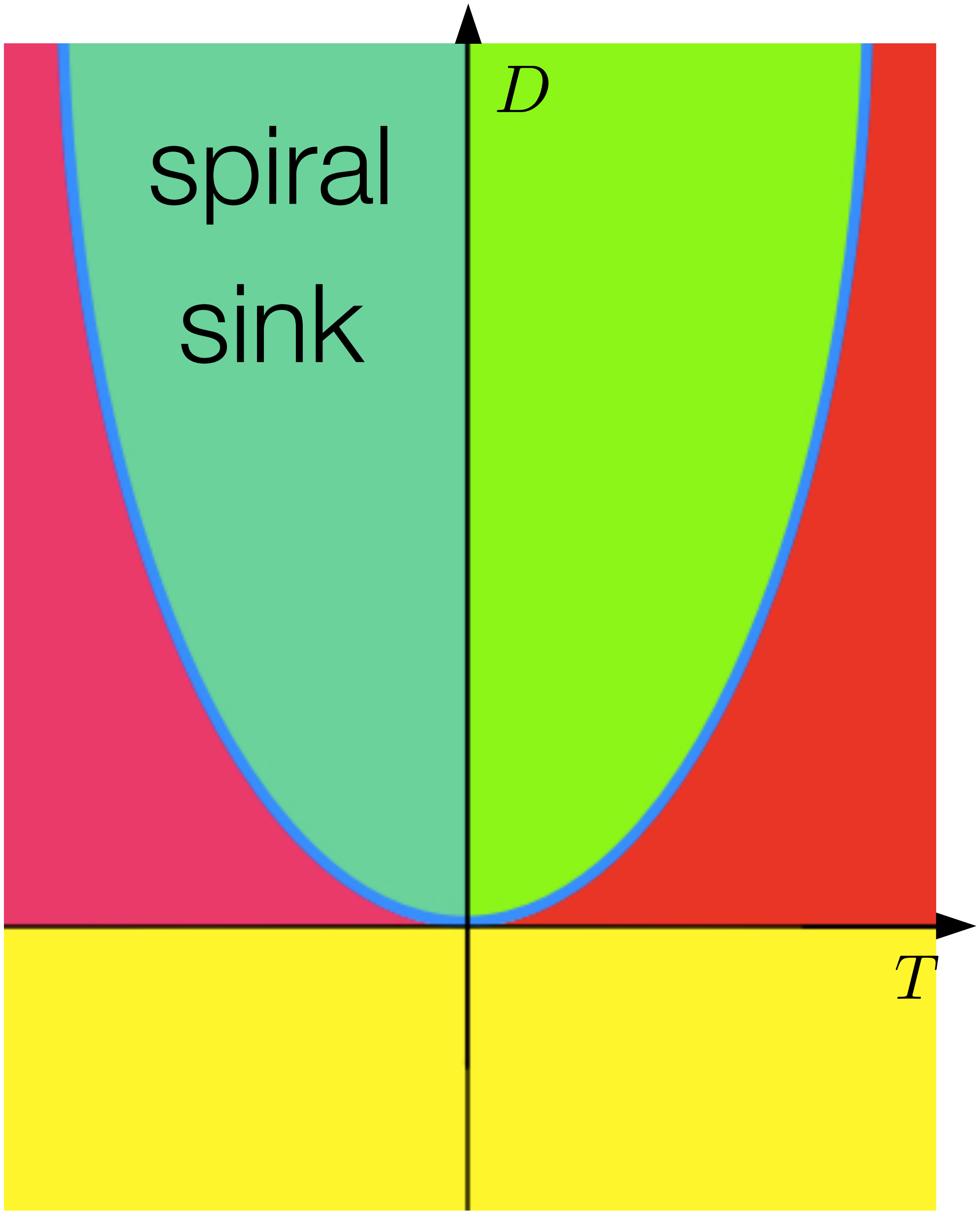
$$T^2 - 4D > 0$$

$$D < 0$$



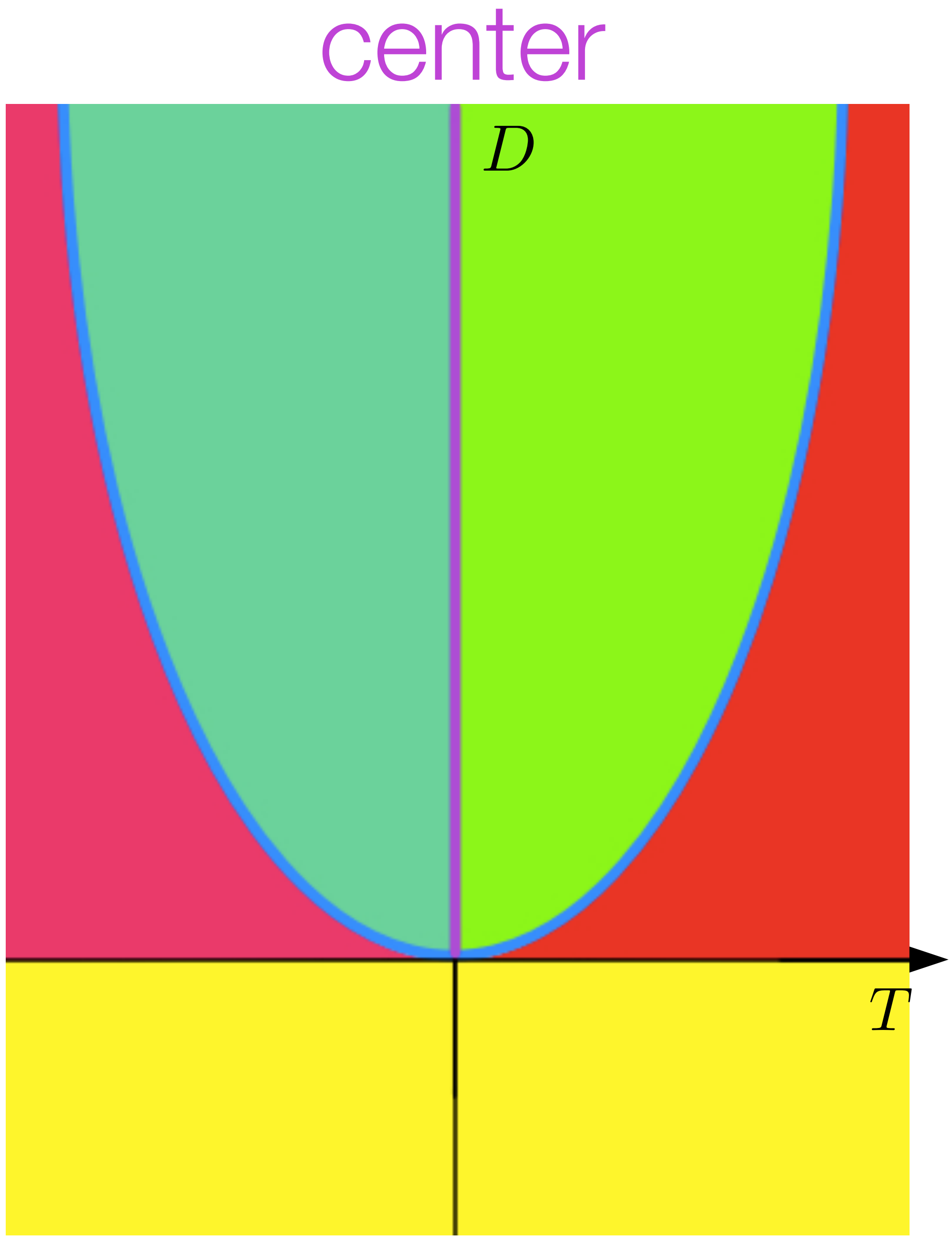
$$T^2 - 4D < 0$$

$$T > 0$$



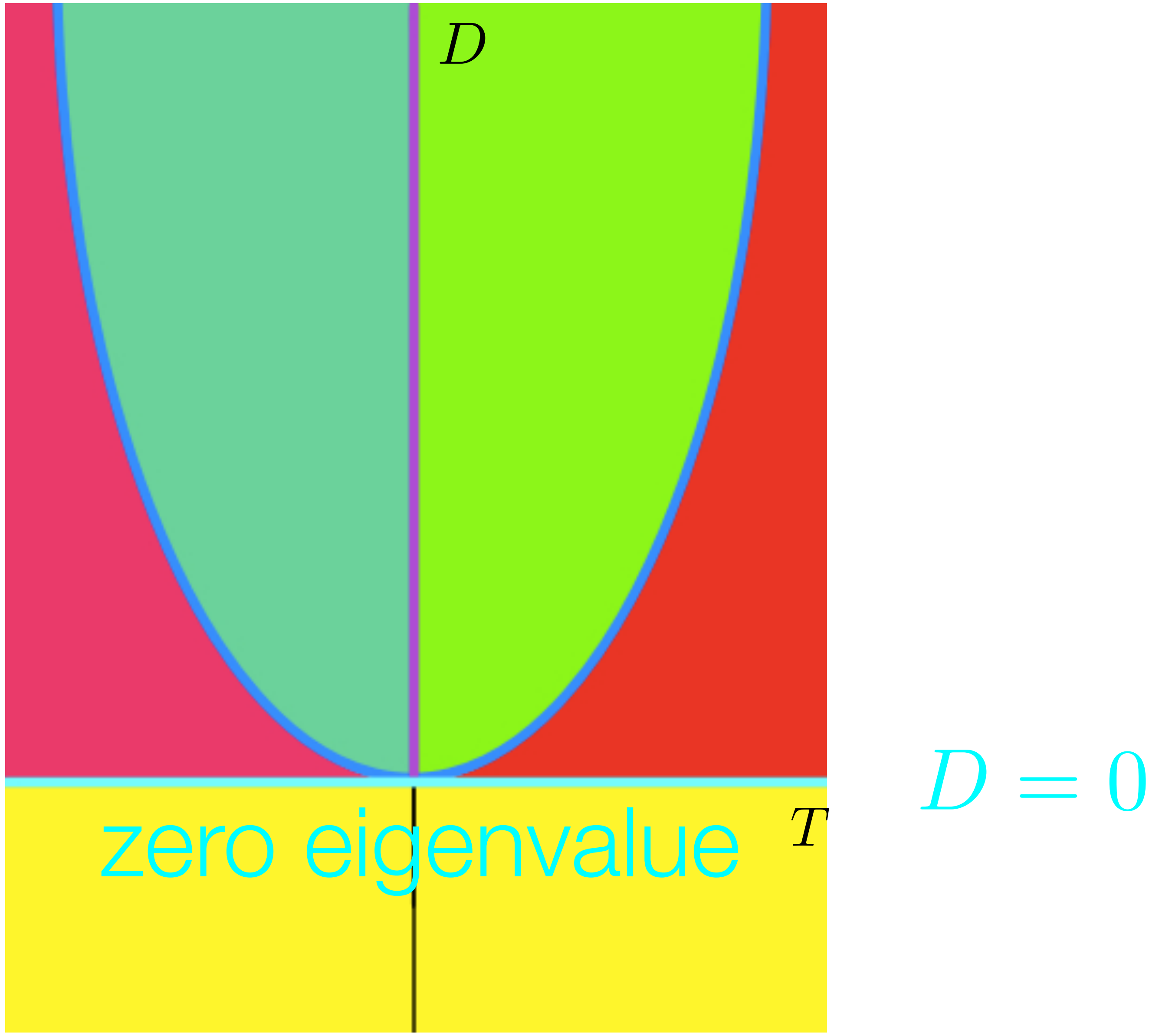
$$T^2 - 4D < 0$$

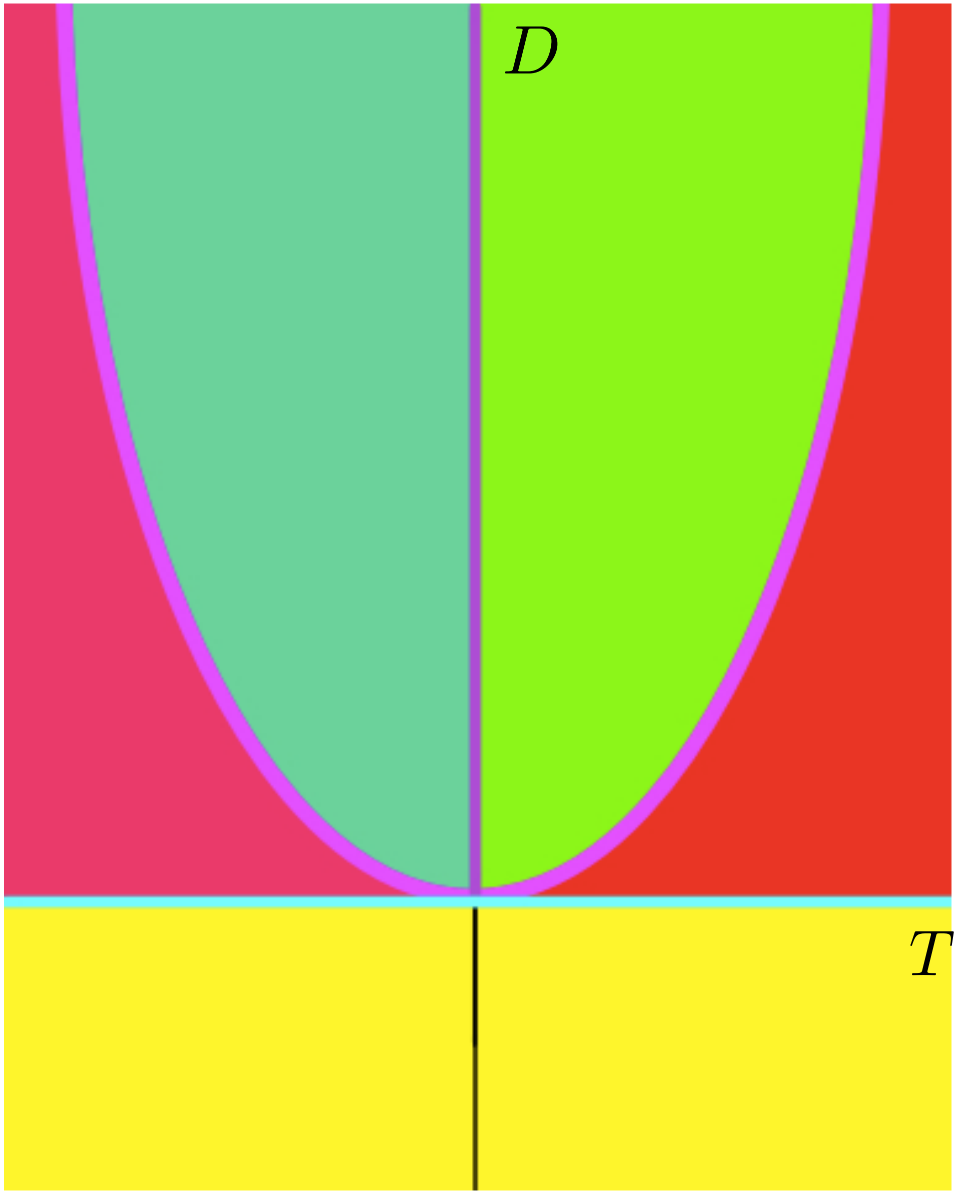
$$T < 0$$



$$T^2 - 4D < 0$$

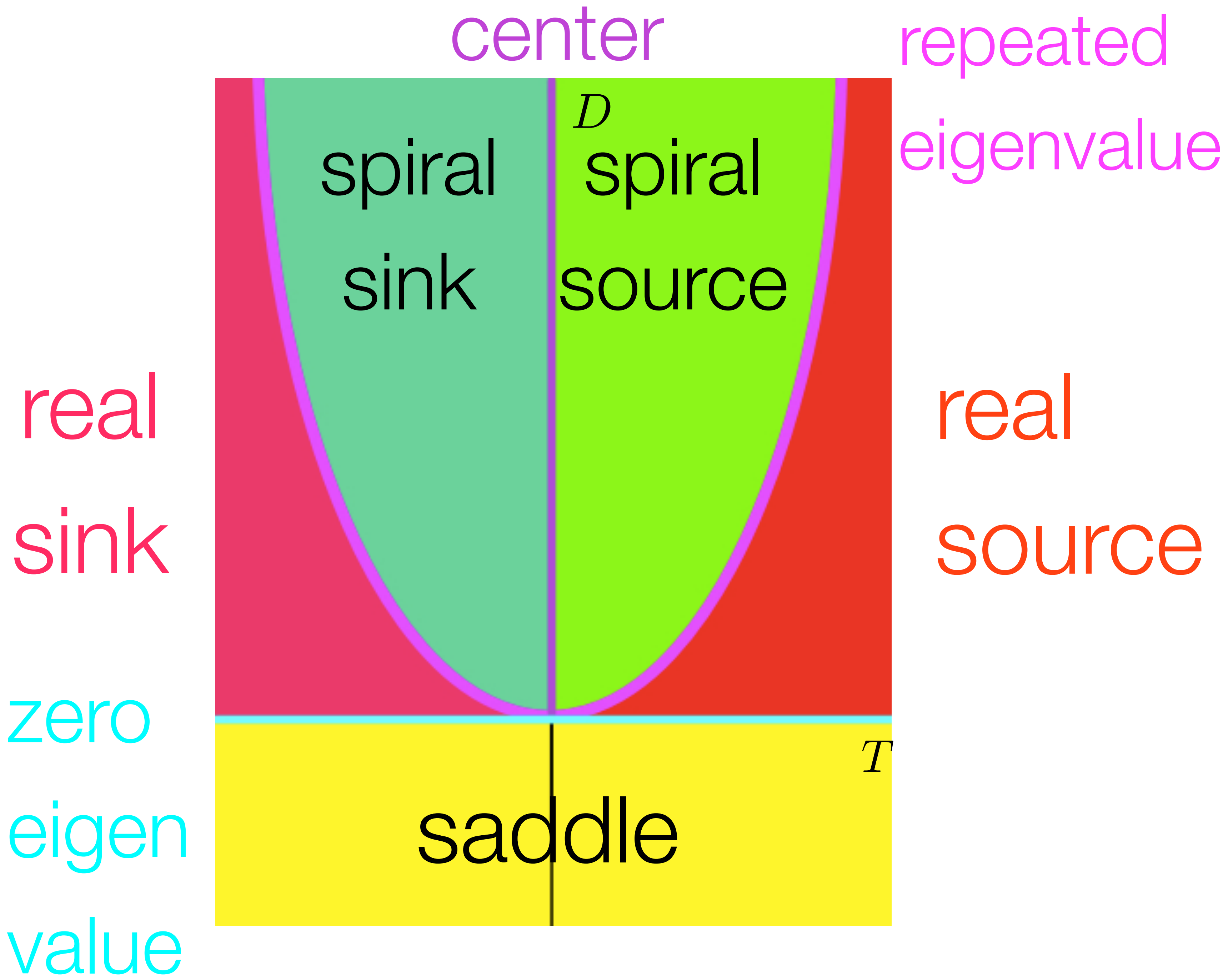
$$T = 0$$

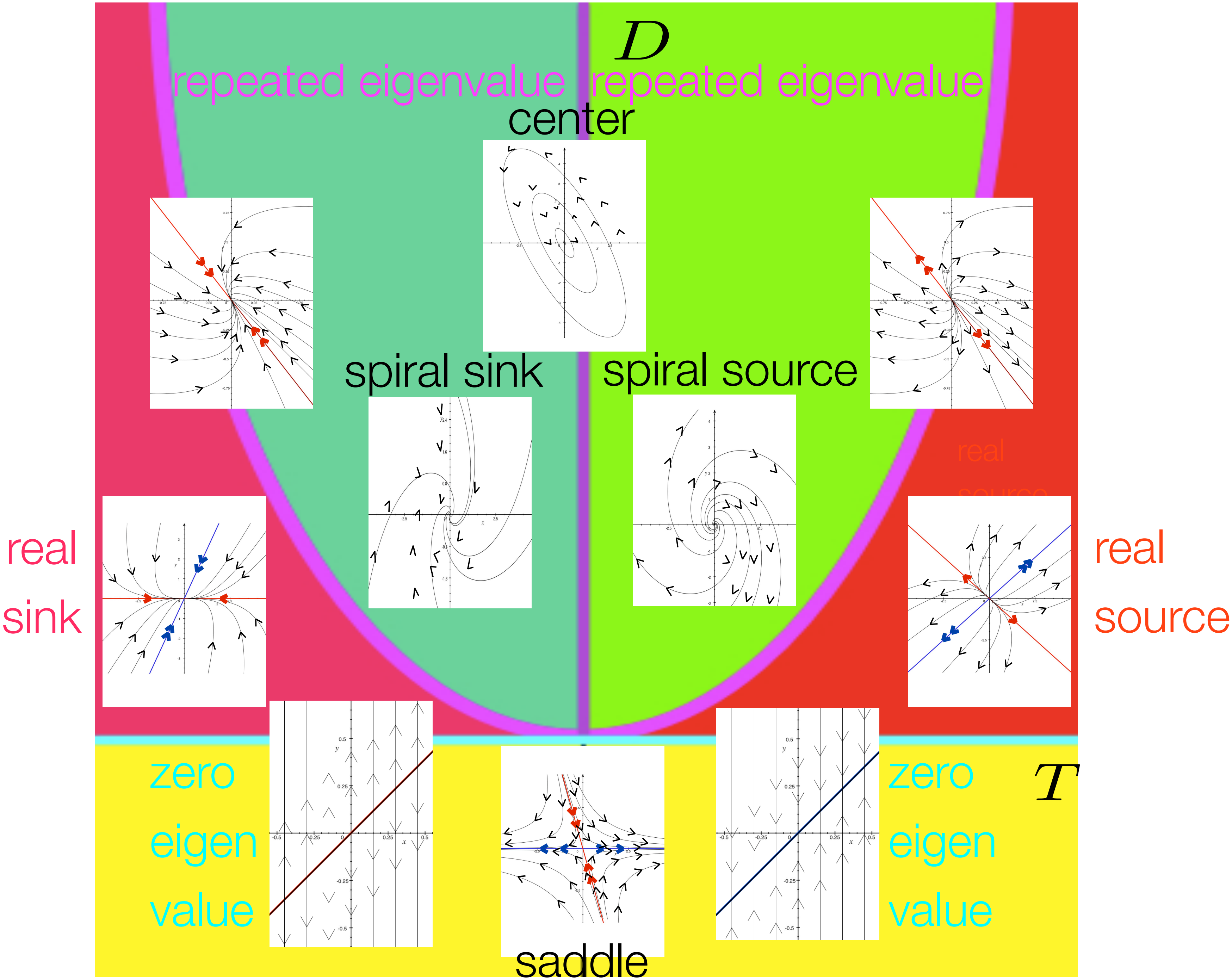

D
T
zero eigenvalue
$D = 0$



$T^2 - 4D = 0$

repeated eigenvalue


center
repeated
eigenvalue
D
spiral
sink
spiral
source
real
sink
real
source
zero
eigen
value
T
saddle


D
repeated eigenvalue
repeated eigenvalue
center
spiral sink
spiral source
real sink
real source
zero eigen value
zero eigen value
T
saddle

- Example:

$$\vec{\mathbf{Y}}' = \begin{pmatrix} 0 & a \\ -1 & -1 \end{pmatrix} \vec{\mathbf{Y}}$$