Day 17: July 27th

- **Chapter: 4.1 Harmonic Oscillators**
  - Homework:
    - Page 393 #1-13 odd, 20, 21, 23.
- **Chapter: 4.2 Sinusoidal Forcing**
  - Homework:
    - Page 406 #1, 5, 9, 11, 13, 17.
- **LAB 2 due tomorrow, July 28th**
- **LAB 3 posted today, due Aug. 4th**

Recall, Harmonic Oscillator

$$my'' + by' + ky = 0$$

$$my'' = -by' - ky$$

$$m = 1 \qquad b \geq 0 \qquad k \geq 0$$

$$\vec{\mathbf{Y}}' = \begin{pmatrix} 0 & 1 \\ -k & -b \end{pmatrix} \vec{\mathbf{Y}}$$

$$my'' + by' + ky = F(t)$$

$$y'' + by' + ky = 0$$

$$y = e^{st}$$

$$s_1, s_2$$

$$y_1(t) = e^{s_1 t} \qquad y_2(t) = e^{s_2 t}$$

$$y(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

- Examples: $y'' + by' + ky = 0$

- <u>Examples</u>:

$$y'' + 4y' + 3y = 0$$

$$y(t) = k_1 e^{-t} + k_2 e^{-3t}$$

- <u>Examples</u>:

$$y'' - y = 0$$

$$y(t) = k_1 e^{t} + k_2 e^{-t}$$

- <u>Examples</u>:

$$y'' + y = 0$$

$$y(t) = k_1 \cos t + k_2 \sin t$$

- Examples:

$$y'' + 3y = 0 \qquad y(0) = 0 \qquad y'(0) = 1$$

- Examples

$$y'' + 4y' + 4y = 0 \quad y(t) = k_1 e^{-2t} + k_2 t e^{-2t}$$

- Examples

$$y'' + 3y' = 0 \qquad y(t) = k_1 + k_2 e^{-3t}$$

- Examples

$$y'' = 0 \qquad y(t) = k_1 + k_2 t$$

2nd order NH eq

$$y'' + by' + ky = F(t)$$

$$y' = v$$

$$v' = -bv - ky + F(t)$$

non-autonomous

so solutions may cross

$$y'' + by' + ky = F(t)$$

damping spring constant forcing term

$$y'' + by' + ky = \alpha\cos(\omega t)$$

recall first order case

$$y' + ay = F(t)$$

$$y' + ay = 0$$

$$y(t) = ke^{-at}$$

guess $y_p(t)$

general solution $y_p(t) + ke^{-at}$

now second order case

$$y'' + by' + ky = F(t)$$

$$y'' + by' + ky = 0$$

we can find general sol of H

$$y_H(t) = k_1 y_1(t) + k_2 y_2(t)$$

guess $y_p(t)$

- Examples:

$$y'' + 4y' + 3y = e^{-2t}$$

$$y'' + 4y' + 3y = 0$$

$$y_H(t) = k_1 e^{-t} + k_2 e^{-3t}$$

guess: $$y_p(t) = Ae^{-2t}$$

$$y_p(t) = -e^{-2t}$$

$$y(t) = k_1 e^{-t} + k_2 e^{-3t} - e^{-2t}$$

- Examples:

$$y'' + y' = e^{-t}$$

$$y_H(t) = k_1 + k_2 e^{-t}$$

guess: $$y_p(t) = Ae^{-t}$$

guess: $$y_p(t) = Ate^{-t}$$

$$y(t) = k_1 + k_2 e^{-t} - te^{-t}$$

Periodic Forcing

$$y'' + by' + ky = F \cdot \cos(\omega t)$$

$F =$ amplitude of forcing

$\frac{2\pi}{\omega} =$ period of forcing

$\frac{\omega}{2\pi} =$ frequency

assume $F = 1, k = 2$

Damped Case

$$y'' + by' + 2y = \cos(\omega t)$$

- Expect $b > 0$

- Example

$$y'' + 3y' + 2y = \cos(t)$$

consider instead

$$y'' + 3y' + 2y = e^{\mathrm{i}t}$$

guess

$$y_p(t) = Ae^{\mathrm{i}t}$$

then use real part

complex solution

use real part

$$y'' + 3y' + 2y = \cos(t)$$

$$k_1 e^{-2t} + k_2 e^{-t} + y_p(t)$$

$$k_1 e^{-2t} + k_2 e^{-t} + \frac{1}{10}\cos t + \frac{3}{10}\sin t$$

steady state solution

all solutions approach steady state

General Case

$$y'' + by' + ky = \cos(\omega t)$$

GS of H approaches 0

PS of NH on form $\alpha\cos(\omega t) + \beta\sin(\omega t)$

GS of NH

GS of H + $\alpha\cos(\omega t) + \beta\sin(\omega t)$

$\to 0$ steady state

- <u>Proof</u>:

roots:

so solutions may cross

so solutions may cross