Day 19: July 29th

- **Chapter: 4.3 Undamped Forcing & Resonance**
- **Chapter: 5.1 Equilibrium Point Analysis**
- Homework:
  - Page 466 #1, 3, 7, 9, 11, 17, 23, 25.
- **Chapter: 5.2 Qualitative Analysis**
- Homework:
  - Page 481 #1-11 odd #17-21 odd
- **Midterm 3, Monday Aug. 2nd**
  - Chapters: 3,4 and 5.

Non-linear Systems

$$\frac{dx}{dt} = F(x, y)$$

$$\frac{dy}{dt} = G(x, y)$$

Generally very hard to solve

- Two techniques:
  - Linearization
  - Nullclines

- <u>Example</u>: Competing species

  Populations that compete for the same resources

$$\frac{dx}{dt} = x\left(\mathbf{1} - \frac{\boldsymbol{x}}{\boldsymbol{M}}\right) - a\frac{y}{M})$$

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{1}{M}x\left(M - x - ay\right) \\ \dfrac{dy}{dt} = \dfrac{1}{N}y\left(N - y - bx\right) \end{cases}$$

- Example: Competing species

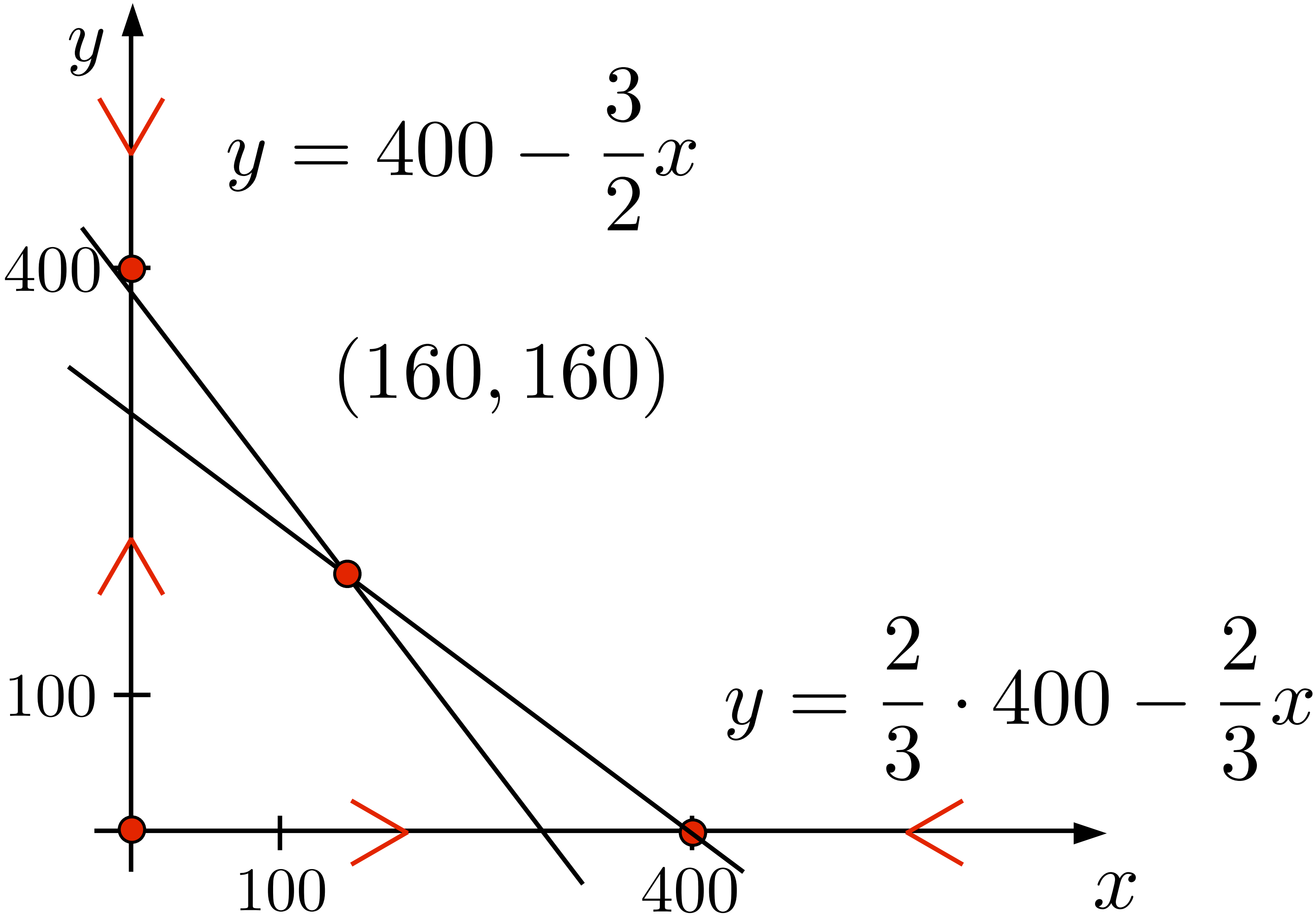
$$\begin{cases} y' = \frac{1}{400} y\left(400 - y - \frac{3}{2}x\right) \\ x' = \frac{1}{400} x\left(400 - x - \frac{3}{2}y\right) \end{cases}$$

$$y = 0 \qquad x = 0$$

$$y = 400 - \frac{3}{2}x \qquad y = \frac{2}{3} \cdot 400 - \frac{2}{3}x$$

- Phase Lines:

- Phase Plane: Other Equilibrium Points



Equilibrium Points

What happens near equilibrium points?

We can “linearize” differential equations near equilibrium points

Jacobian Matrix

Near an equilibrium point a nonlinear system is "almost" linear

Find Jacobian matrix and evaluate at the equilibrium point

$$\mathbf{J} = \begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix}$$

$$\vec{\mathbf{Y}}' = \mathbf{J} \cdot \vec{\mathbf{Y}}$$

- Example:

$$x' = -x \qquad = F(x,y)$$

$$y' = -4x^3 + y \quad = G(x,y)$$

$$\mathbf{J} = \begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix}$$

Equilibrium point: $(0,0)$

Linearized System: $\vec{\mathbf{Y}}' = \mathbf{J} \cdot \vec{\mathbf{Y}}$

Evaluated at $(0,0)$

- <u>Example</u>:

$$x^{'} = -x \qquad\qquad = F(x,y)$$

$$y^{'} = -4x^3 + y = G(x,y)$$

$$\mathbf{J} = \begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix}$$

$$\mathbf{J} = \begin{pmatrix} -1 & 0 \\ -12x^2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\vec{\mathbf{Y}}^{'} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \vec{\mathbf{Y}}$$

- Example:

$$x' = -x + y^2 = F(x,y)$$
$$y' = x^2 + y = G(x,y)$$

Equilibrium point: $(0,0)$

$$\mathbf{J} = \begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix} = \begin{pmatrix} -1 & 2y \\ 2x & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Evaluated at $(0,0)$

- Example:

$$x' = -x + y^2 = F(x,y)$$
$$y' = x^2 + y = G(x,y)$$

Linearized System:

$$\vec{\mathbf{Y}}' = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \vec{\mathbf{Y}}$$

- Example:

$$x' = -x + y^2 = F(x, y)$$
$$y' = x^2 + y = G(x, y)$$

Equilibrium point: $(1, -1)$

$$\mathbf{J} = \begin{pmatrix} \frac{\delta F}{\delta x} & \frac{\delta F}{\delta y} \\ \frac{\delta G}{\delta x} & \frac{\delta G}{\delta y} \end{pmatrix} = \begin{pmatrix} -1 & 2y \\ 2x & 1 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix}$$

Evaluated at $(1, -1)$

- Example:

$$x' = -x + y^2 = F(x,y)$$
$$y' = x^2 + y = G(x,y)$$

Equilibrium point: $(1,-1)$

Linearized System:

$$\vec{\mathbf{Y}}' = \begin{pmatrix} -1 & -2 \\ 2 & 1 \end{pmatrix} \vec{\mathbf{Y}}$$