Day 2: June 29th

- Homework

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  1.3 Page 48 #1, 7, 9, 11, 13, 15, 17.

Modeling

- Differential Equations
  - Assumptions
  - Specify variables and parameters
  - Write out equations
- Solve
  - Analytical (old)
  - Qualitatively
  - Numerically

Modeling Population Growth

- Assume: Rate of growth of P is prop to P if P is "small".
- Assume: "Carrying capacity".

$$P \text{ constant if } P = N$$

$$P \downarrow \text{ if } P > N$$

$$P \uparrow \text{ if } P < N$$

- Predict: Population at time t

- Variables:
  - t = time
    = indipendent variable
  - P = P(t)
    = Population at time t
    = dependent variable
- Parameters:
  - N = Capacity (parameter)
  - k = proportionality constant (parameter)

Model

$$\frac{dP}{dt} = k \cdot P \cdot \Big(? \Big)$$

$\Big(? \Big) \approx 1$ if P small

$\Big(? \Big) = 0$ if P=N

$\Big(? \Big) < 0$ if P>N

Model

$$\frac{dP}{dt} = k \cdot P \cdot \Big(? \Big)$$

assume $N = 1$

$$\frac{dP}{dt} = k \cdot P \cdot \Big(1 - P \Big)$$

Model

$$\frac{dP}{dt} = k \cdot P \cdot \Big(? \Big)$$

Generally:

$$\frac{dP}{dt} = k \cdot P \cdot \left(1 - \frac{P}{N}\right)$$

Example

$$\frac{dP}{dt} = 0.3 \cdot P\left(1 - \frac{P}{500}\right)$$

- Solve
  - Equilibrium solutions
  - Analytical approach
  - Qualitative approach

Equilibrium solutions

First Rule of Baldur:

$$\text{set } \frac{dP}{dt} = 0$$

$$\cancel{\frac{dP}{dt}}\, 0 = 0.3 \cdot P \cdot \left(1 - \frac{P}{500}\right)$$

Analytical approach

Separate and Integrate

Let's do it

Initial value problem: $P(0) = 100$

Initial value problem: $P(0) = 500$

Qualitative approach

Slope Field

Conclusions:

$$\left.\begin{array}{r} P_0 = 0 \\ P_0 = 500 \end{array}\right\}$$ $P(t)$ is constant
equilibrium solutions

$P_0 > 0$ $P(t)$ tends to 500

$P(t) \downarrow$ if $P_0 > 500$

$P(t) \uparrow$ if $P_0 < 500$