

Day 3: June 30th

- Homework

6/30 1.4 Page 63 #1, 3, 15.

Differential Equations

- An equation for a missing function involving the derivative of the function

- $\frac{dy}{dt} = \sin(t) \cdot y^2$

- $\frac{dy}{dt} = \sin(t)$

- $\frac{dy}{dt} = y^2$

First Order Equations

$$\frac{dy}{dt} = F(y, t)$$

Second Order Equations

$$\frac{d^2y}{dt^2} = F\left(y, \frac{dy}{dt}, t\right)$$

System of ODEs

$$F' = -\beta F + R \cdot F$$

$$R' = \alpha R - R \cdot F$$

$$\frac{dx}{dt} = F_1(x, y, z, t)$$

$$\frac{dy}{dt} = F_2(x, y, z, t)$$

$$\frac{dz}{dt} = F_3(x, y, z, t)$$

Solutions

Solution to $\frac{dy}{dt} = F(y, t)$

is a function that “works”.

Example: $\frac{dy}{dt} = \sin(t)$

Solutions: $y_1(t) = -\cos(t)$

$$y_2(t) = -\cos(t) + 5$$

$$y_3(t) = -\cos(t) + k$$

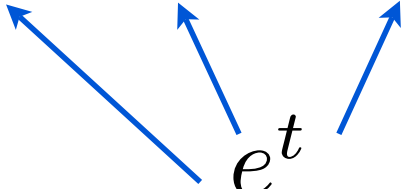
NOT $y_4(t) = \cos(t)$

Verify Solutions

You can always check solution

Example: $\frac{dy}{dt} = y(1 - y)$

Joe Schmoe
hands you
solution

$$y(t) = \frac{e^t}{e^t - 1}$$


Plug in and see if “=” holds

You solve using integration

Special Cases

what to do?

• case 1: $\frac{dy}{dt} = F(t)$

integrate

• case 2: $\frac{dy}{dt} = F(y)$

separate
autonomous
and integrate

• case 3: $\frac{dy}{dt} = F(y, t)$

depends!

General Solution

“Family” of functions that may be used to solve any initial value problem

i.e. all possible solutions

Initial value problem: $y(t_0) = y_0$

Examples

$$\frac{dy}{dt} = y - 3 = F(y)$$

$$\frac{dy}{dt} = \sin(t) \cdot y^2 \quad y(0) = 1 \quad y(0) = 0$$

$$\frac{dy}{dt} = 1 + y^2 = F(y) \quad y(0) = \alpha$$

$$\frac{dy}{dt} = y^2$$

Homework

6/29 1.2 Page 33 #1, 3, 5, 7, 9, 15, 23.

1.3 Page 48 #1, 7, 9, 11, 13, 15, 17.