Day 4: July 1st

- Homework

  7/1 1.5 Page 73 #1, 3, 5, 7, 13.

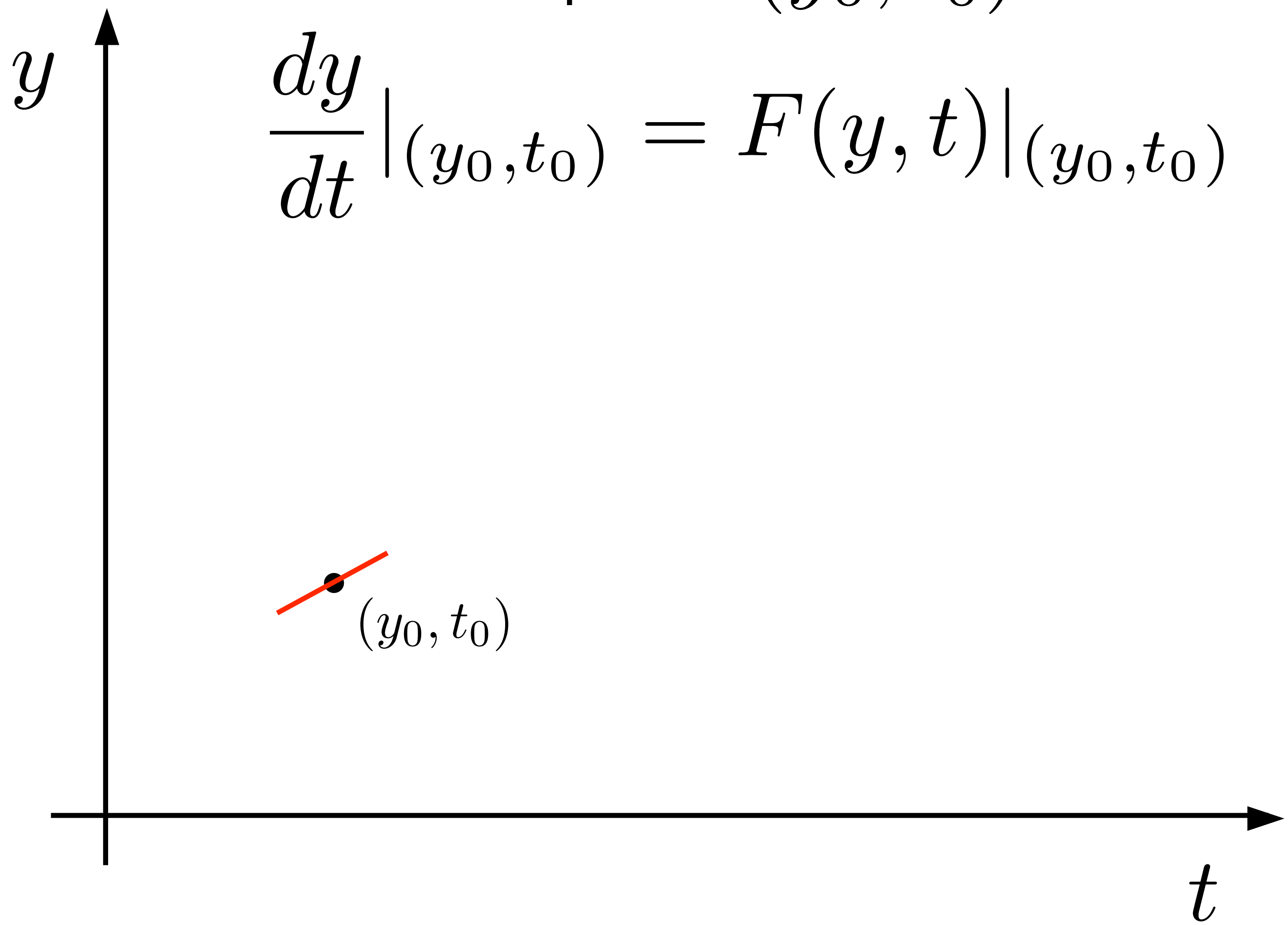
- http://math.bu.edu/people/baldur/MA226.html

Slope Field

$$\frac{dy}{dt} = F(y,t)$$

slope at (y_0, t_0)

$$\frac{dy}{dt}\Big|_{(y_0,t_0)} = F(y,t)\Big|_{(y_0,t_0)}$$



Slope Field

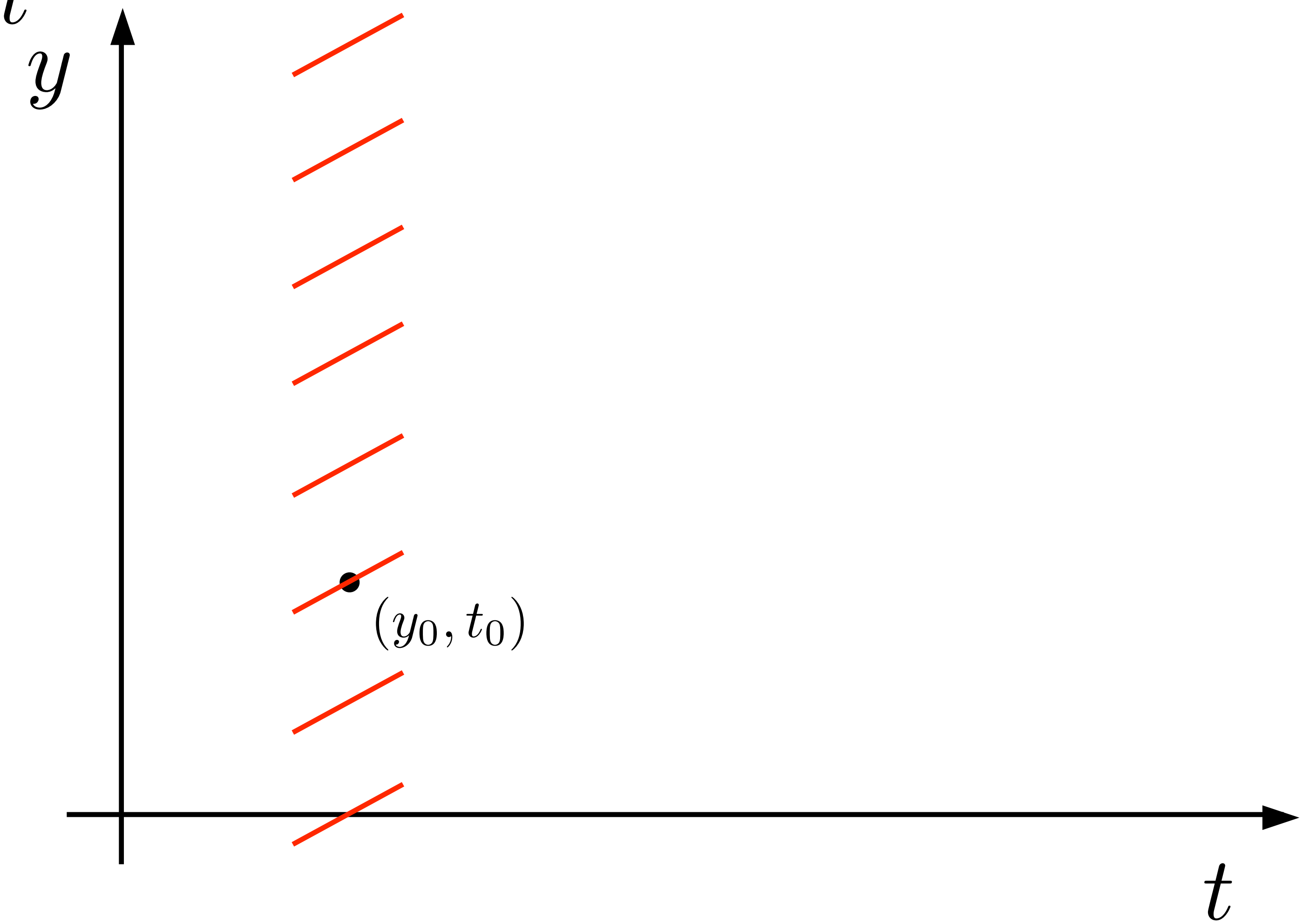
Special Cases

- case 1: $$\frac{dy}{dt} = F(t)$$
- case 2: $$\frac{dy}{dt} = F(y)$$

Slope Field

$$\frac{dy}{dt} = F(t)$$

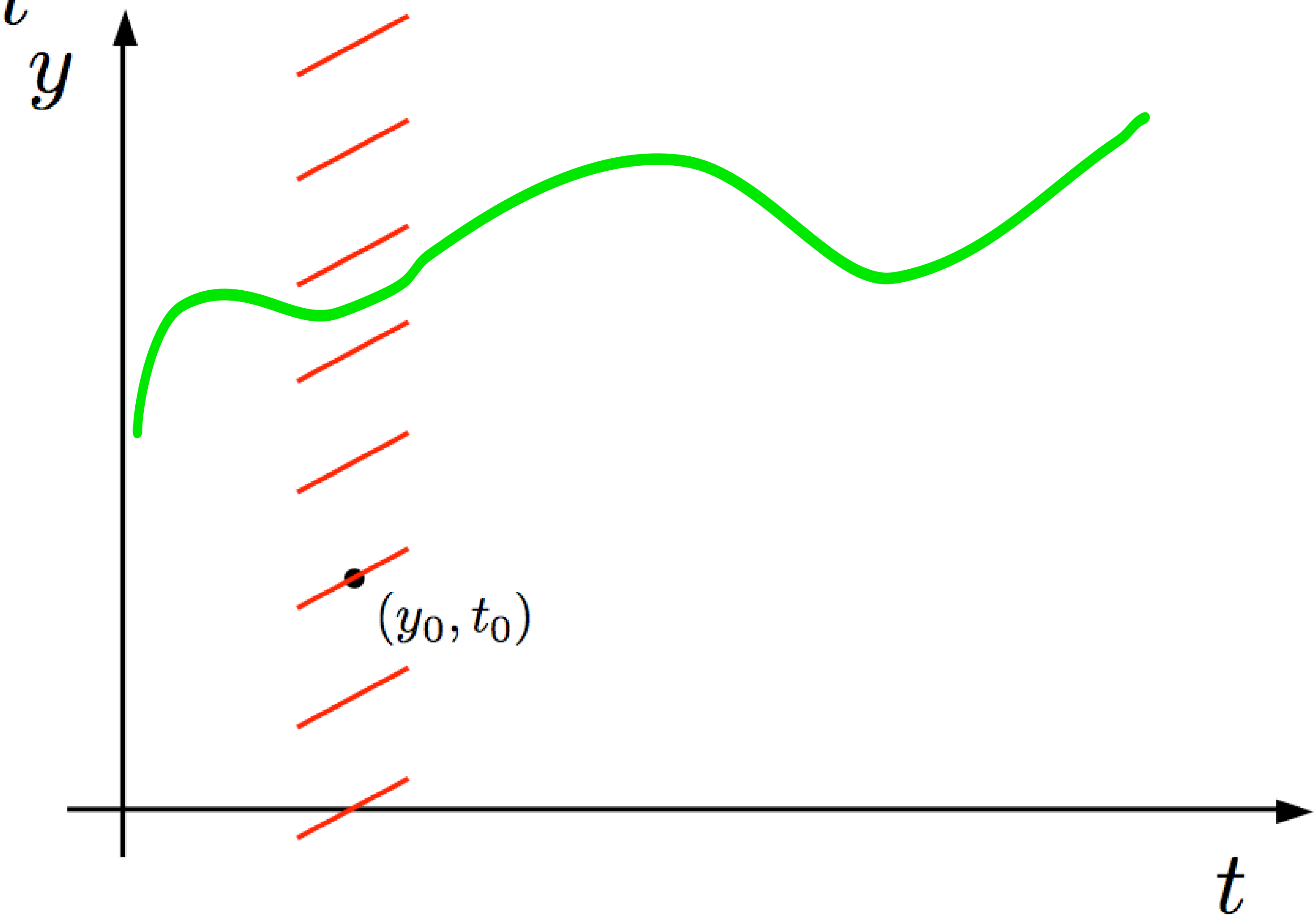
slope at (y_0, t_0)



Slope Field

$$\frac{dy}{dt} = F(t)$$

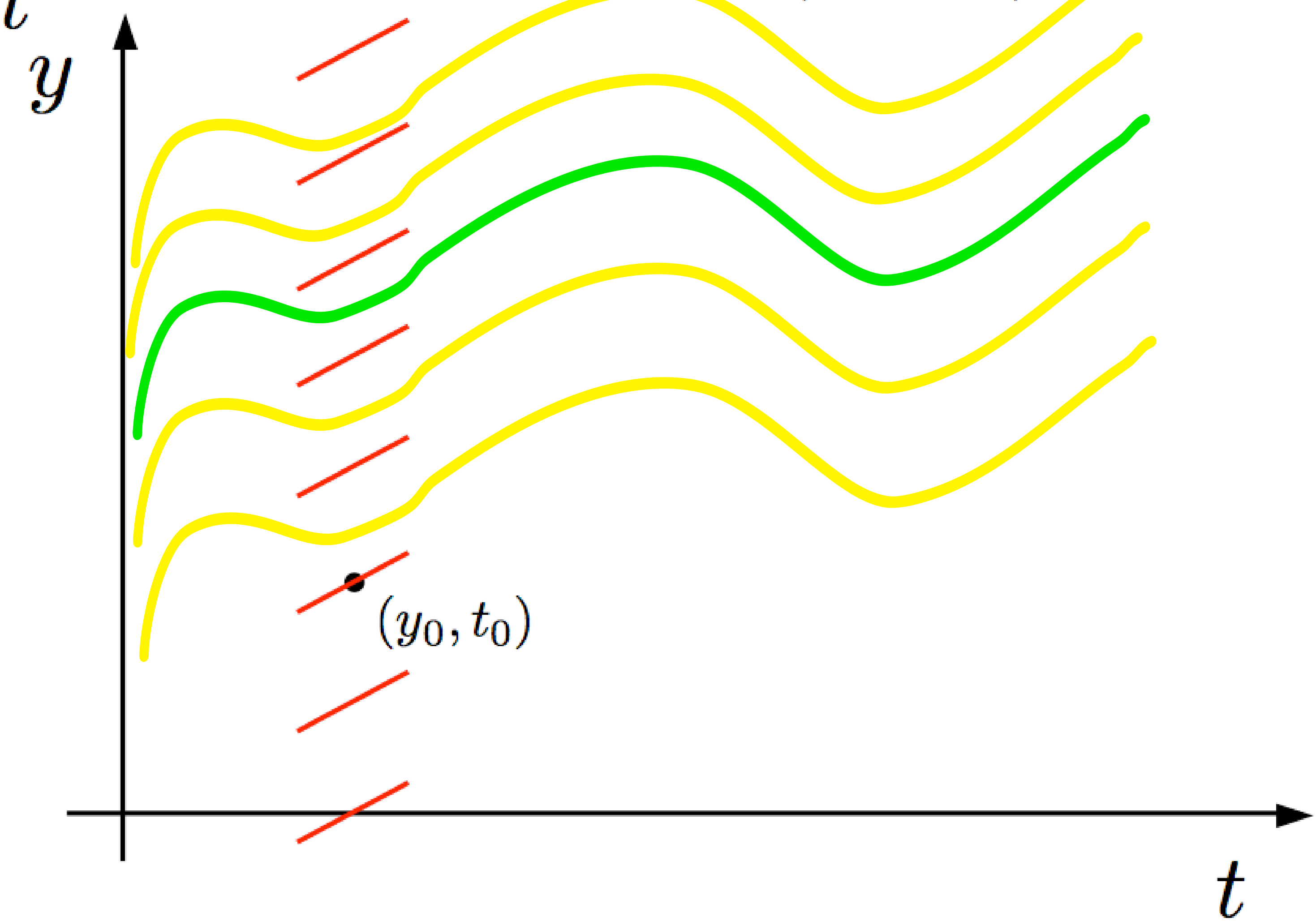
slope at (y_0, t_0)



Slope Field

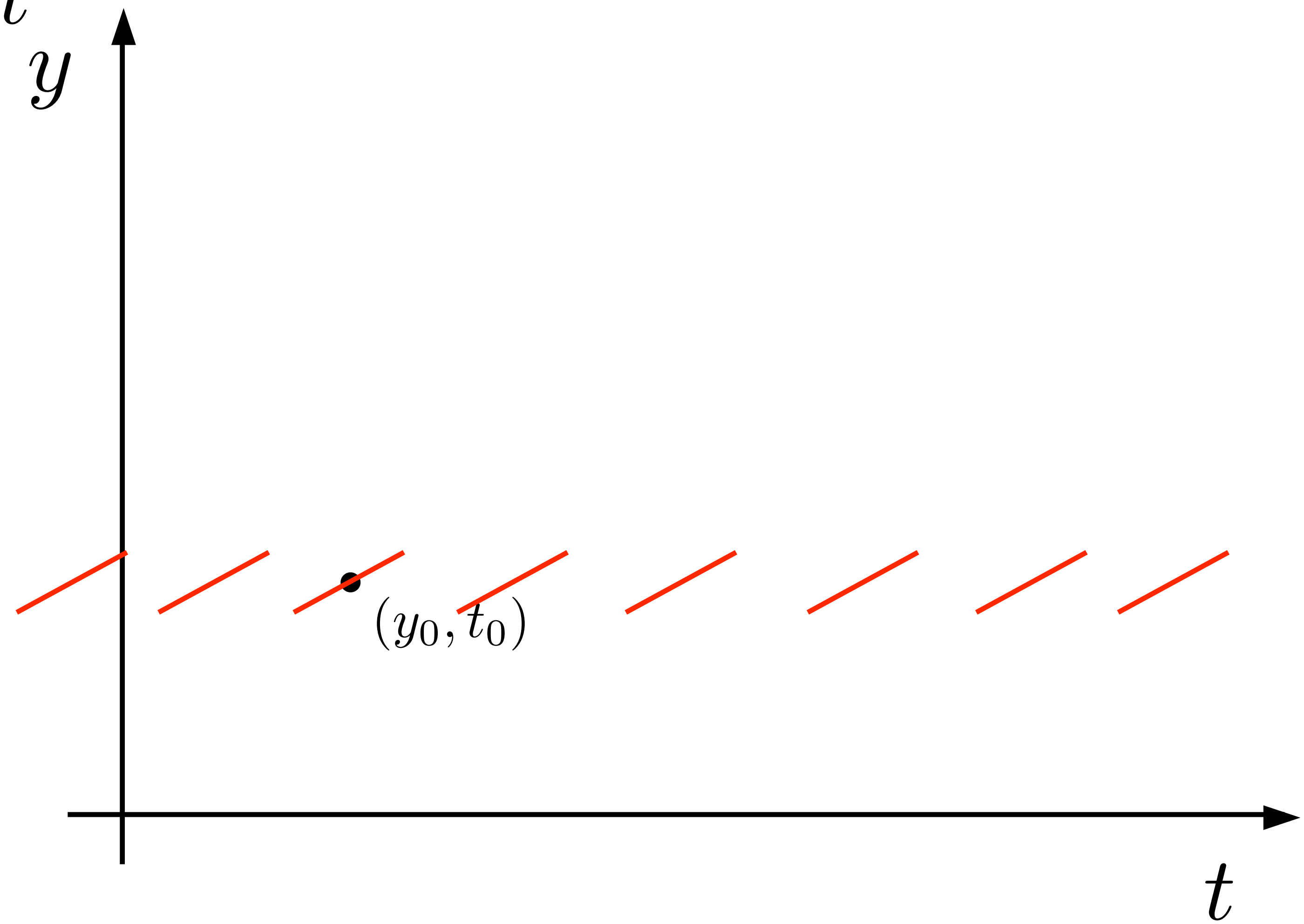
$$\frac{dy}{dt} = F(t)$$

slope at (y_0, t_0)



Slope Field

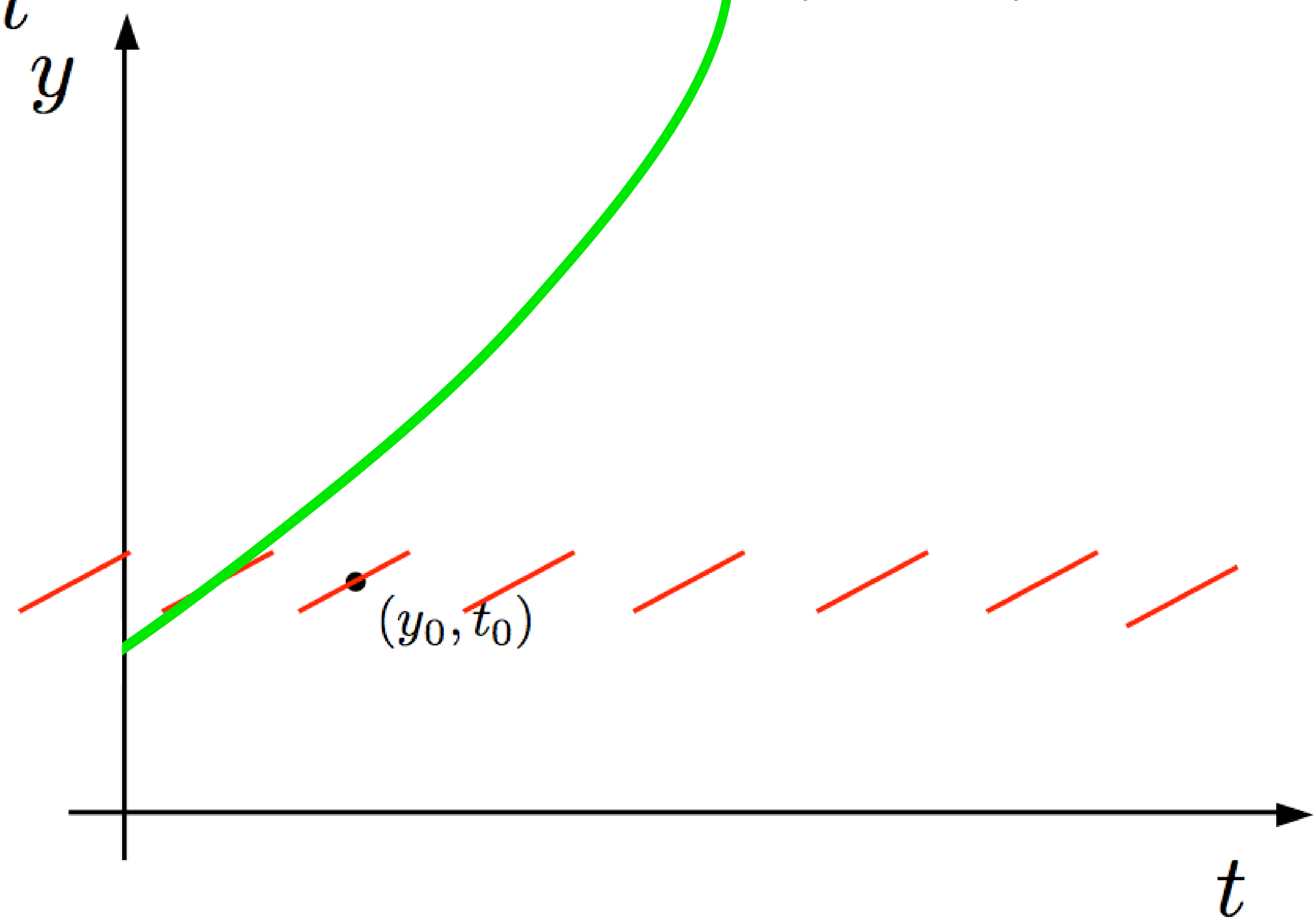
$$\frac{dy}{dt} = F(y)$$ slope at (y_0, t_0)



Slope Field

$$\frac{dy}{dt} = F(y)$$

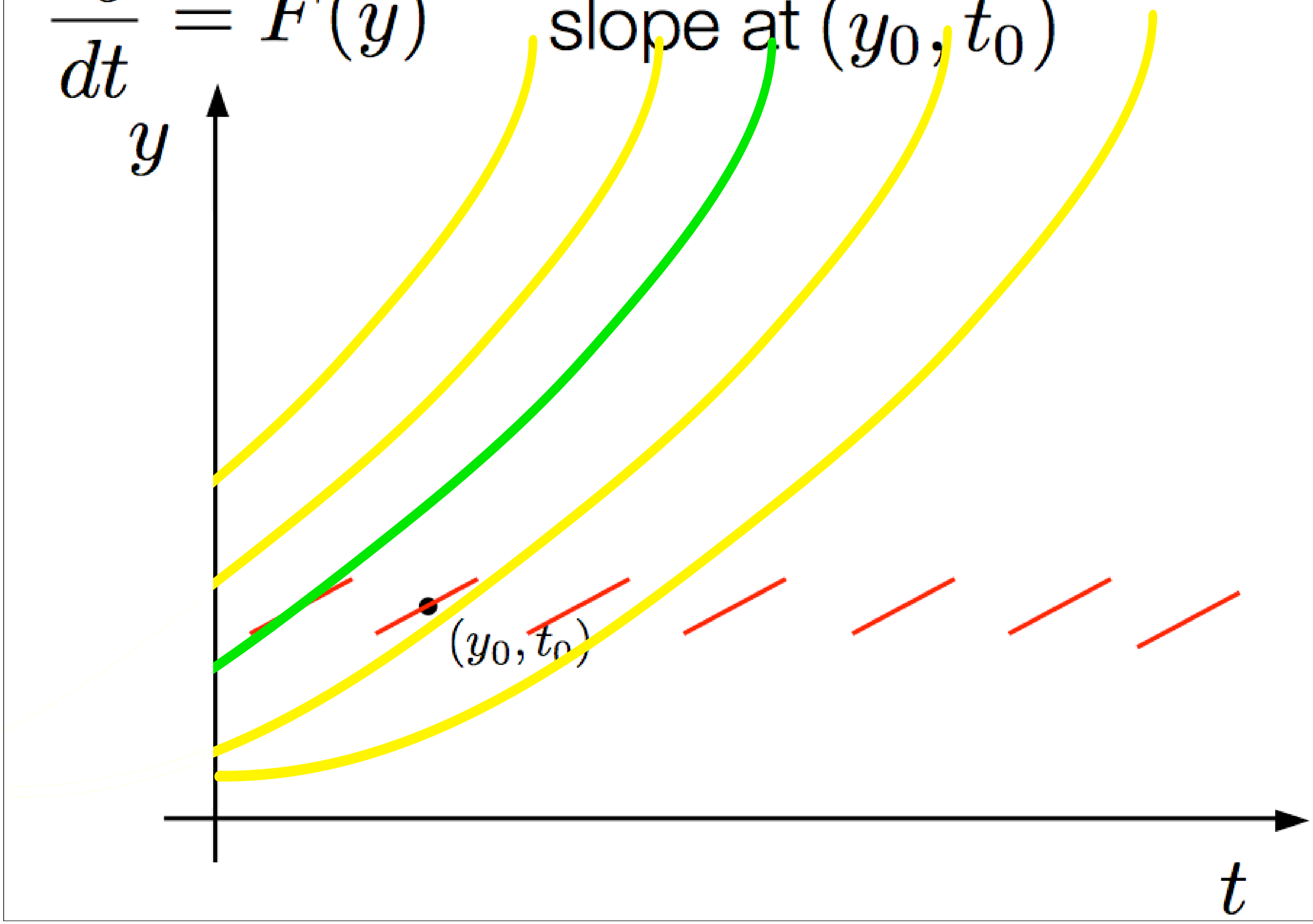
slope at (y_0, t_0)



Slope Field

$$\frac{dy}{dt} = F(y)$$

slope at (y_0, t_0)



Difficult Integrals

$$\frac{dy}{dt} = \sin^2(y) \qquad \frac{dy}{dt} = e^y \sin^2(y)$$

Equilibrium points: $y = p \cdot \pi \quad p \in Z$

$$\int \frac{1}{\sin^2(y)} dy = \int dt$$

$$\int \frac{1}{e^y \sin^2(y)} dy = \int dt$$

Difficult Integrals

$$\frac{dy}{dt} = \sin^2(y) \qquad \frac{dy}{dt} = e^y \sin^2(y)$$

Equilibrium points: $y = p \cdot \pi \quad p \in Z$

Analytical method: … if you are Albert Einstein

Qualitative method: … yes please

slope field

Numerical Method

Euler's Method:

Solve $\dfrac{dy}{dt} = F(y,t)$ numerically

$$y(t_0) = y_0$$

Idea: Travel along slope field with step size Δt

How to move from (t_n, y_n) to (t_{n+1}, y_{n+1})?

$$F(t_n, y_n) = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_{n+1} - y_n}{t_{n+1} - t_n} = \frac{y_{n+1} - y_n}{\Delta t}$$

$$F(t_n, y_n) = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_{n+1} - y_n}{t_{n+1} - t_n} = \frac{y_{n+1} - y_n}{\Delta t}$$

$$F(t_n, y_n) = \frac{y_{n+1} - y_n}{\Delta t}$$

$$F(t_n, y_n) \cdot \Delta t = y_{n+1} - y_n$$

$$y_n + F(t_n, y_n) \cdot \Delta t = y_{n+1}$$

Euler's Method:

$$y_{n+1} = y_n + F(t_n, y_n) \cdot \Delta t$$

$$t_{n+1} = t_n + \Delta t$$

Example: $\dfrac{dy}{dt} = 2t$

solution: $y(t) = t^2 + c$

initial condition: $y(0) = \alpha$

$$\begin{array}{c} y(0) \\ \| \\ 0^2 + c \\ \| \\ c \end{array} \qquad \alpha = c$$

General solution: $y(t) = t^2 + \alpha$

$$y(t) = t^2 + \alpha$$

$$y(0) = 1$$

correct solution

$$y(t) = t^2 + 1 \qquad y(1) = 2$$

Approximate $y(1)$ via Euler

<u>Euler's Method:</u> $\boxed{t_{n+1} = t_n + \Delta t}$

$$y_{n+1} = y_n + F(t_n, y_n) \cdot \Delta t$$

$$\frac{dy}{dt} = F(t, y) = 2t \quad \boxed{y_{n+1} = y_n + 2t_n \cdot \Delta t}$$

$$F(t_n, y_n) = 2t_n$$

Given: $t_0 = 0$, $y_0 = 1$ Pick $\Delta t = 0.1$

Euler's Method

$$\frac{dy}{dt} = 2t$$

Formula:

$$\boxed{\begin{aligned} t_{n+1} &= t_n + \Delta t \\ y_{n+1} &= y_n + 2t_n \cdot \Delta t \end{aligned}}$$

Numerical Error:

We know right answer, how close are we?

Try different step sizes: Δt

Be careful

$$\frac{dy}{dt} = e^t \sin^2(y)$$

Equilibrium points: $y = p \cdot \pi \quad p \in Z$

$t \ll 0 \Rightarrow \dfrac{dy}{dt} \approx 0$ slope zero

$t \gg 0 \Rightarrow \dfrac{dy}{dt} \approx \pm \mathrm{BIG}$ large positive slope
or
large negative slope

Existence and Uniqueness

$$\frac{dy}{dt} = F(y,t)$$

Existence: there is a solution

Uniqueness: there is just one given an initial condition

Common use: solutions can not cross

$F(y,t)$ has to be "nice"