Day 5: July 6th

- **Chapter: 1.5 Existence and Uniqueness.**
- **Chapter: 1.6 Equilibria and Phase Line.**
- Homework:

  1.6 Page 91: #1, 3, 5, 7, 13, 15, 17, 29-35 odd.

Coming up

- **Midterm 1 on Friday, July 9th: Chapter 1**
- **Lab 1 due Thursday next week, July 15th**

Existence and Uniqueness

$$\frac{dy}{dt} = F(y,t)$$

If: Continuously differentiable in y and t, i.e.

F is differentiable in y and t and the derivative is continuous

Then: There exists a unique solution to initial value problem $y(t_0) = y_0$

defined for $t_0 - A < t < t_0 + A$

Existence and Uniqueness

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Result: Solutions can not cross.

Example: Homework. Chapter 1.5: 1,3,5 & 7.

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1. $$\frac{dy}{dt} = f(t, y)$$

$y_1(t) = 3$ for all t is a solution

what can we say about $y(0) = 1$

3. $$\frac{dy}{dt} = f(t, y)$$

$y_1(t) = t + 2$ for all t is a solution

$y_2(t) = -t^2$ for all t is a solution

what can we say about $y(0) = 1$

Example: Homework. Chapter 1.5: 1,3,5 & 7.

$$\frac{dy}{dt} = (y-2)(y-3)y$$

5. what can we say about $y(0) = 4$

7. what can we say about $y(0) = 1$

E & U: Examples

When existence or uniqueness fails

1. $$\frac{dy}{dt} = \frac{y}{t}$$

2. $$\frac{dy}{dt} = \frac{y}{t^2}$$

Look for “bad” points

3. $$\frac{dy}{dt} = \frac{3}{2}y^{\frac{1}{3}}$$

E & U: Examples

1. $\dfrac{dy}{dt} = \dfrac{y}{t}$

Lots of solutions satisfy: $y(0) = 0$

but none satisfy $y(0) = k \neq 0$

E & U: Examples

2. $\dfrac{dy}{dt} = \dfrac{y}{t^2}$ Homework: 1.5 #13

$y(t) = 0$ is an equilibrium solution

E & U: Examples

2. $\dfrac{dy}{dt} = \dfrac{y}{t^2}$ Homework: 1.5 #13

$$y(t) = De^{-\frac{1}{t}}$$

As $t \to 0^+, \ \ y(t) \to 0$

$t \to 0^-, \ y(t) \to \pm\infty$

$y(t) = 0$ is an equilibrium solution

E & U: Examples

3. $\dfrac{dy}{dt} = \dfrac{3}{2} y^{\frac{1}{3}}$ derivative not continuous at 0

$y(t) = 0$ is an equilibrium solution

Lots of solutions satisfy: $y(0) = 0$

Phase Line

Autonomous equations

$$\frac{dy}{dt} = F(y)$$

slopes do not depend on t

sign of F shows whether solutions increase or decrease.

Ex: Logistic equation

$$\frac{dy}{dt} = y(1 - y)$$