Day 6: July 7th

- **Chapter: 1.6 Equilibria and Phase Line.**
- **Chapter: 1.7 Bifurcations**
- Homework:

  1.7 Page 107 #1, 3, 9, 11, 22, 23.

- **Midterm 1 on Friday, July 9th: Chapter 1**

Different Pictures

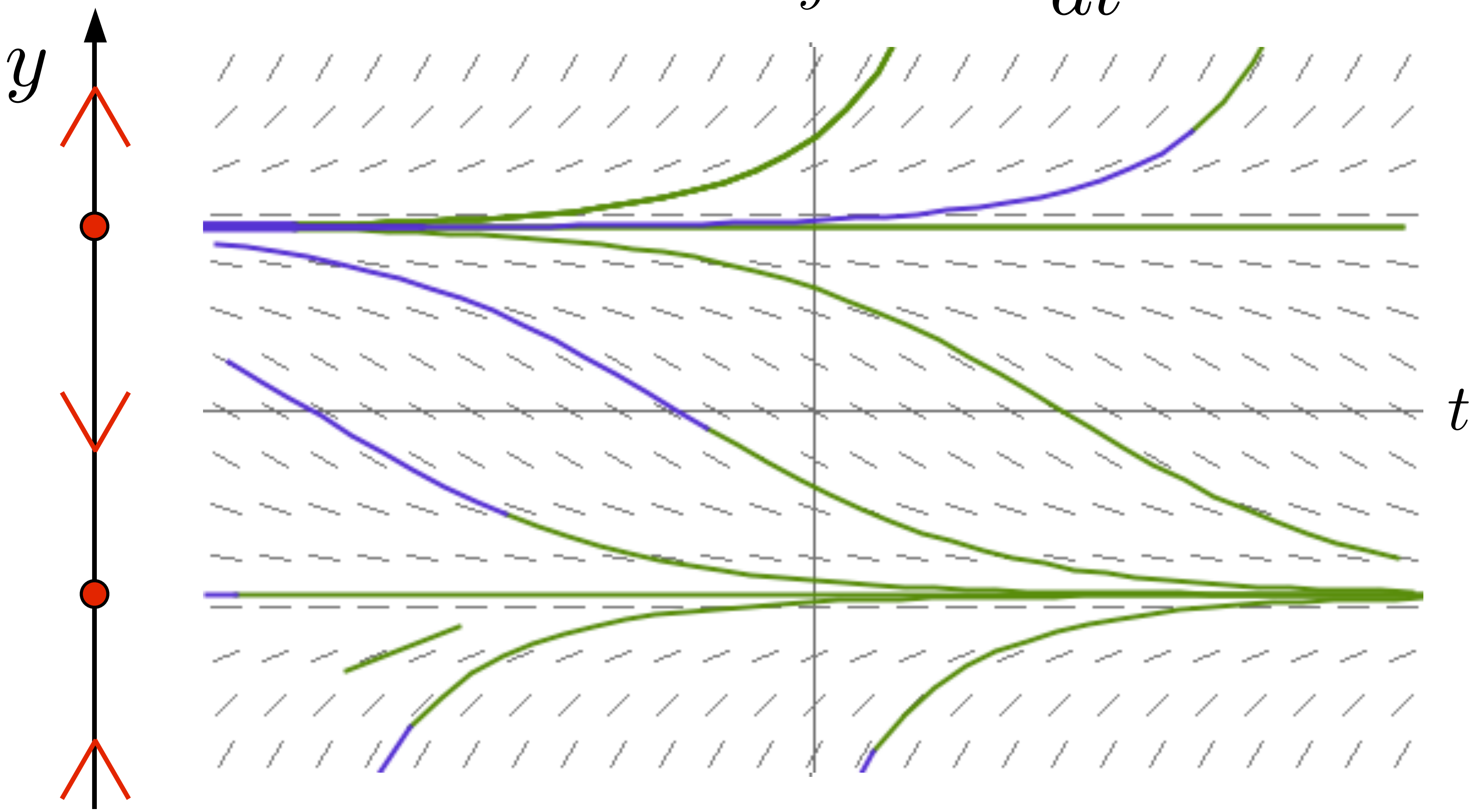
Any differential equation $\dfrac{dy}{dt} = F(y, t)$

- Slope field
- Solution curves

Autonomous equation $\dfrac{dy}{dt} = F(y)$

- Phase line

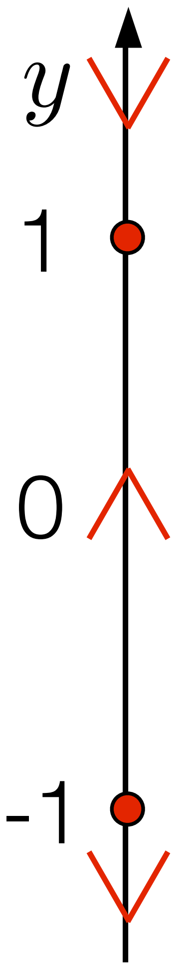
$$\frac{dy}{dt} = F(y)$$



Solutions: $y(t)$

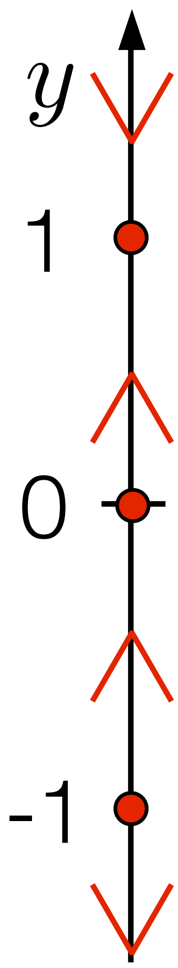
Phase line example

$$\frac{dy}{dt} = 1 - y^2$$



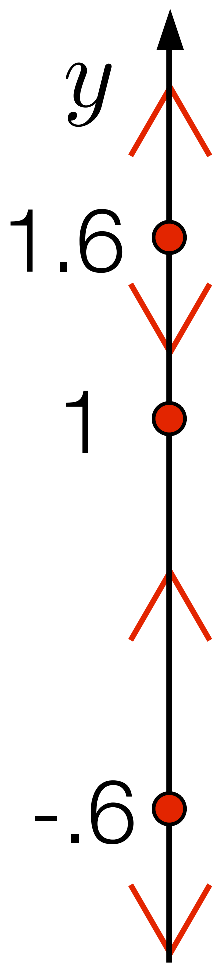
Phase line example

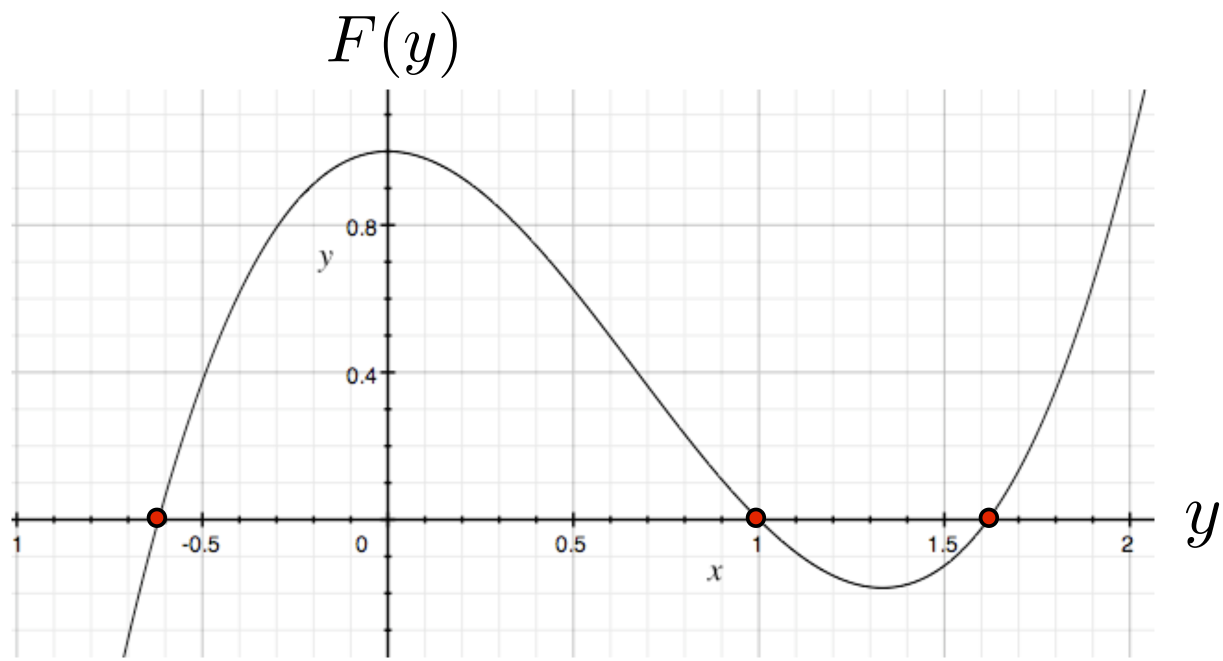
$$\frac{dy}{dt} = y^2(1 - y^2)$$



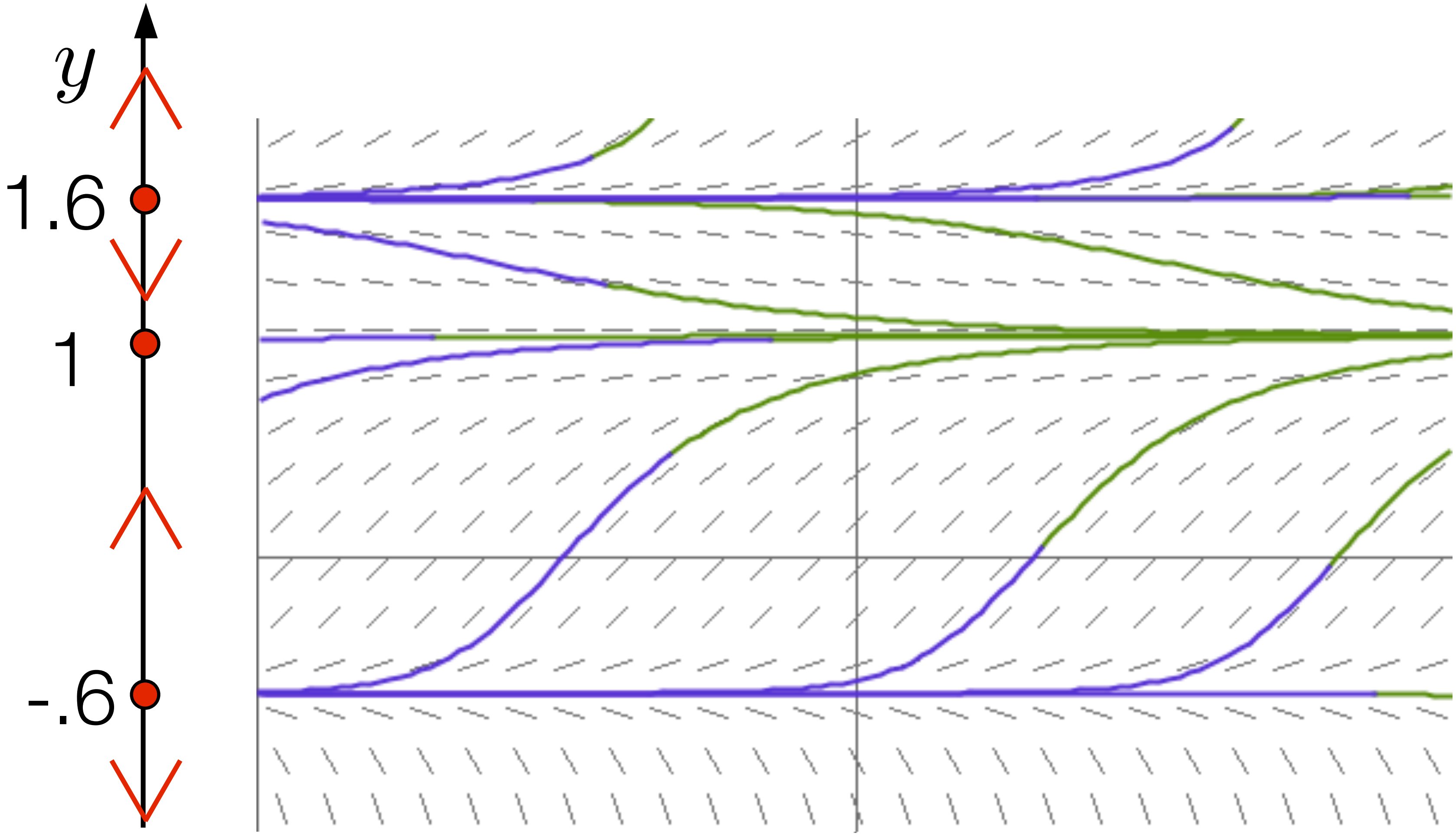
Phase line from $F(y)$ graph

$$\frac{dy}{dt} = F(y)$$

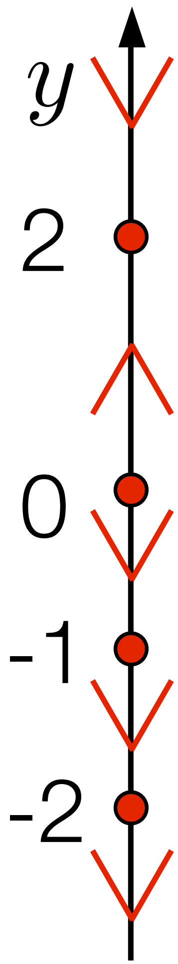




Solutions



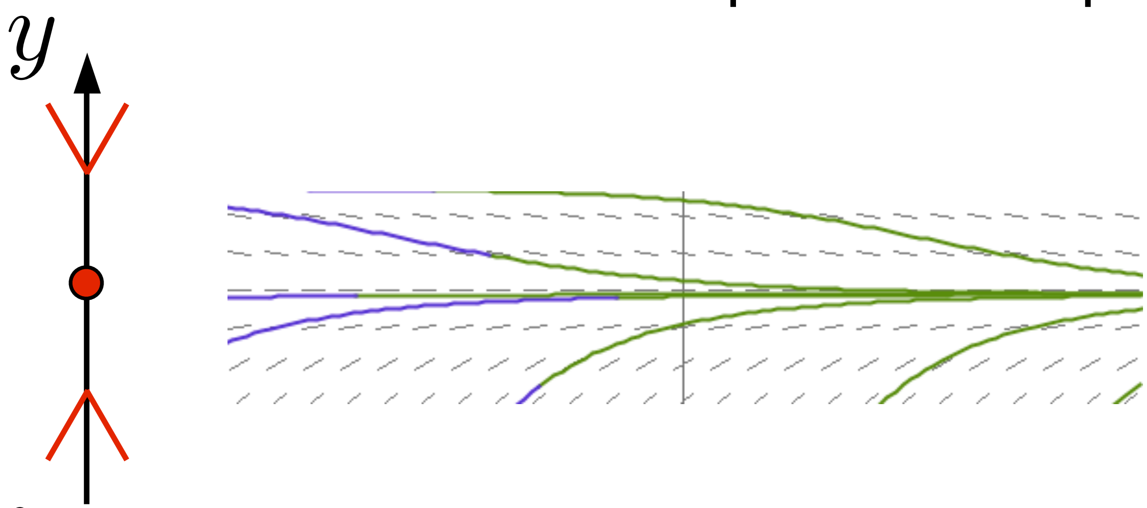
Graph of $F(y)$ from phase line



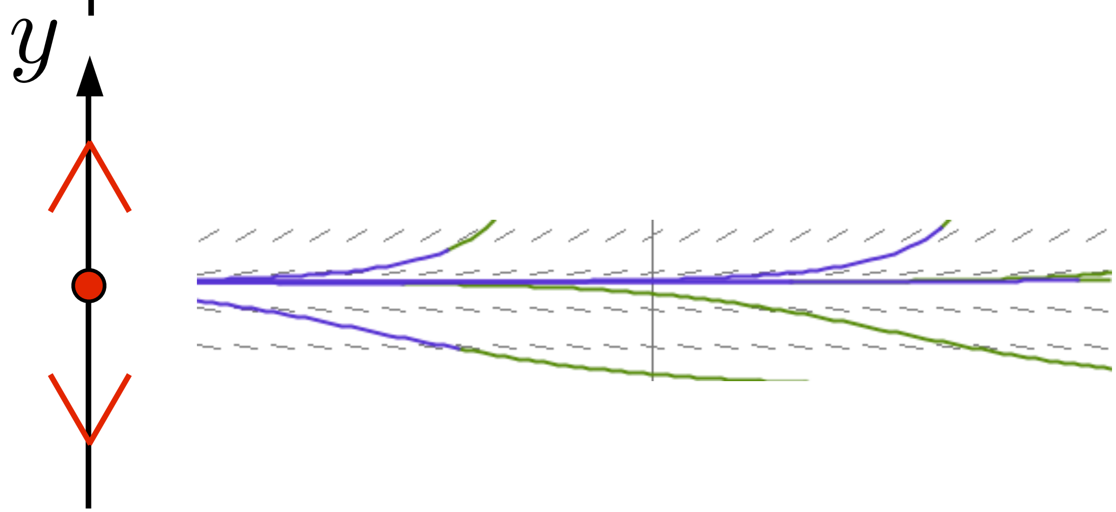
$$F(y) = ?$$

Types of equilibrium solutions

also called equilibrium points



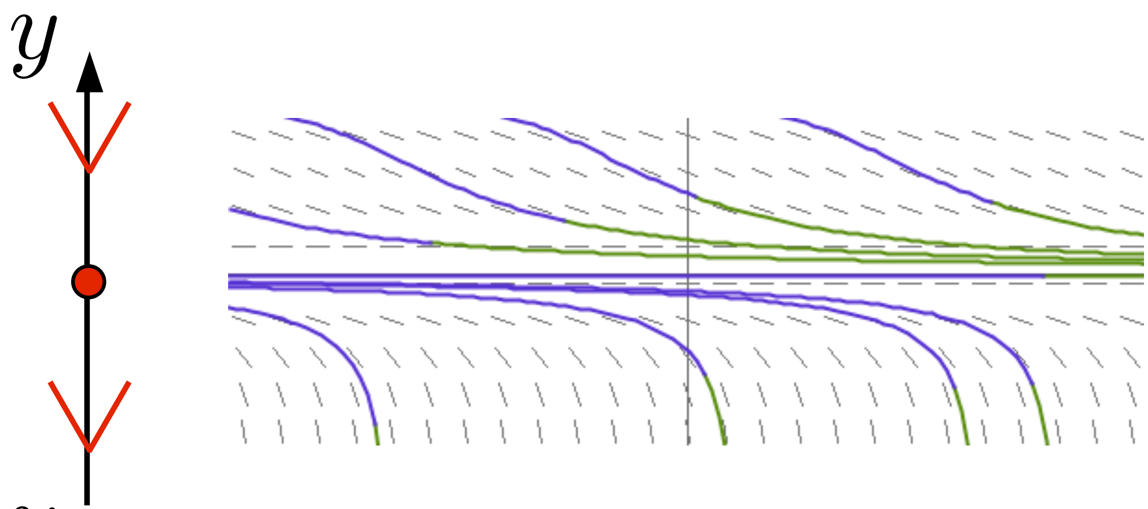
sink



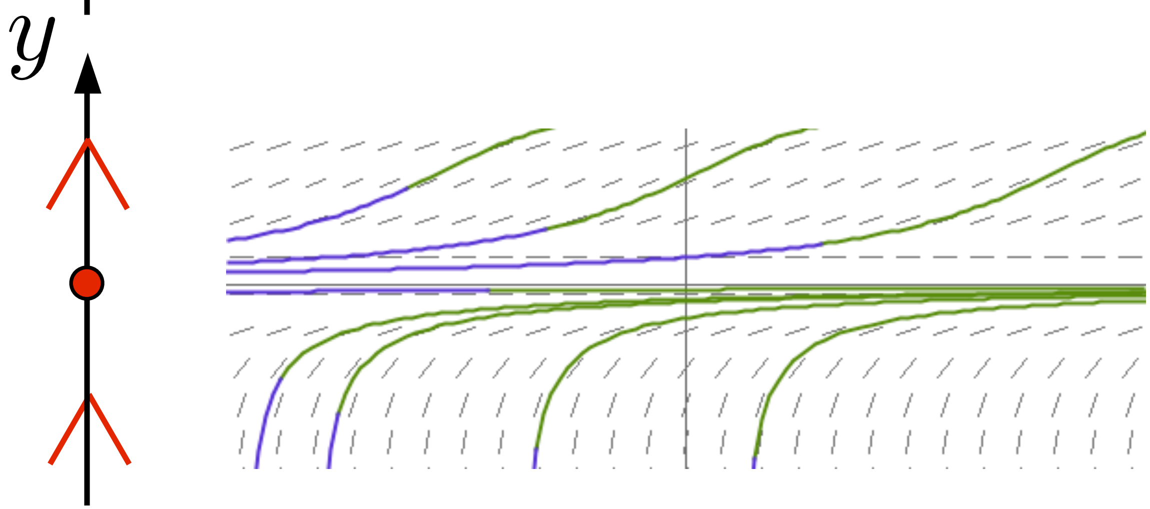
source

Types of equilibrium solutions

neither a sink or a source



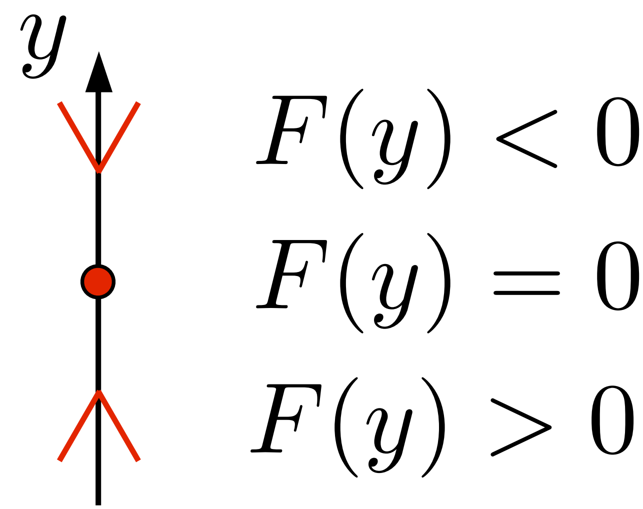
node



node

Finding type of equilibrium point

$$\frac{dy}{dt} = F(y)$$



Note: it is $F(y)$ not $F'(y)$ that determines slope

$$\frac{dy}{dt} = F(y)$$

First Derivative Test

$$\frac{dy}{dt} = F(y)$$

Have equilibrium point $F(y_0) = 0$

$F^{'}(y_0) > 0 \Rightarrow$ y_0 is a source

$F^{'}(y_0) < 0 \Rightarrow$ y_0 is a sink

$F^{'}(y_0) = 0 \Rightarrow$ no info about y_0

Exercise 5 page 91

$$\frac{dy}{dt} = y^2 - 6y - 7$$

Example

$$\frac{dy}{dt} = y^4 - 4y^2$$

1.7: Bifurcations

$$\frac{dy}{dt} = F_a(y)$$

A bifurcation is a BIG change in the over all behavior of solutions as a changes

Example

$$\frac{dy}{dt} = F_a(y) = y^2 + a$$

bifurcation at $a = 0$

Most common bifurcations

$$\frac{dy}{dt} = F_a(y)$$

- number of equilibrium point changes
- type of equilibrium point changes

Bifurcation diagram

Phase lines for different a values plotted to show how behavior of solutions depends on a