Day 7: July 8th

- **Chapter: 1.7 Bifurcations**
- **Chapter: 1.8 Linear Equations**
- Homework:
  - 1.8 Page 123 #1, 3, 7, 9, 13.
  - Chapter 1: Rev Prob: Pg 138 #1, 3, 5, 9, 10, 11, 13, 17, 21, 23, 31, 33, 37, 43, 45.
- **Midterm 1 tomorrow: Chapter 1**

Bifurcations

$$\frac{dy}{dt} = F_a(y)$$

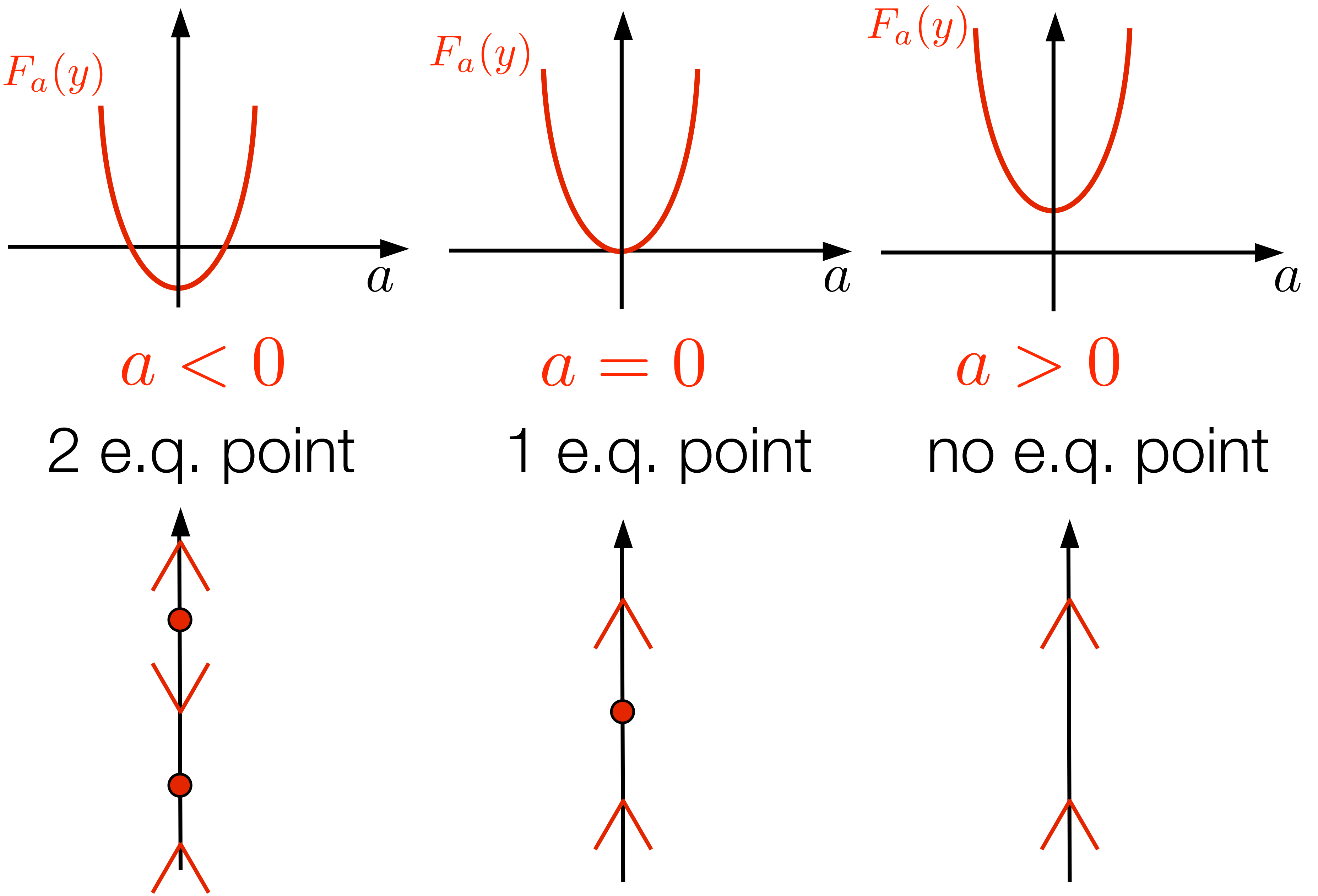
A BIG change in the over all behavior of solutions as a changes, $a \in \mathbb{R}$

Example

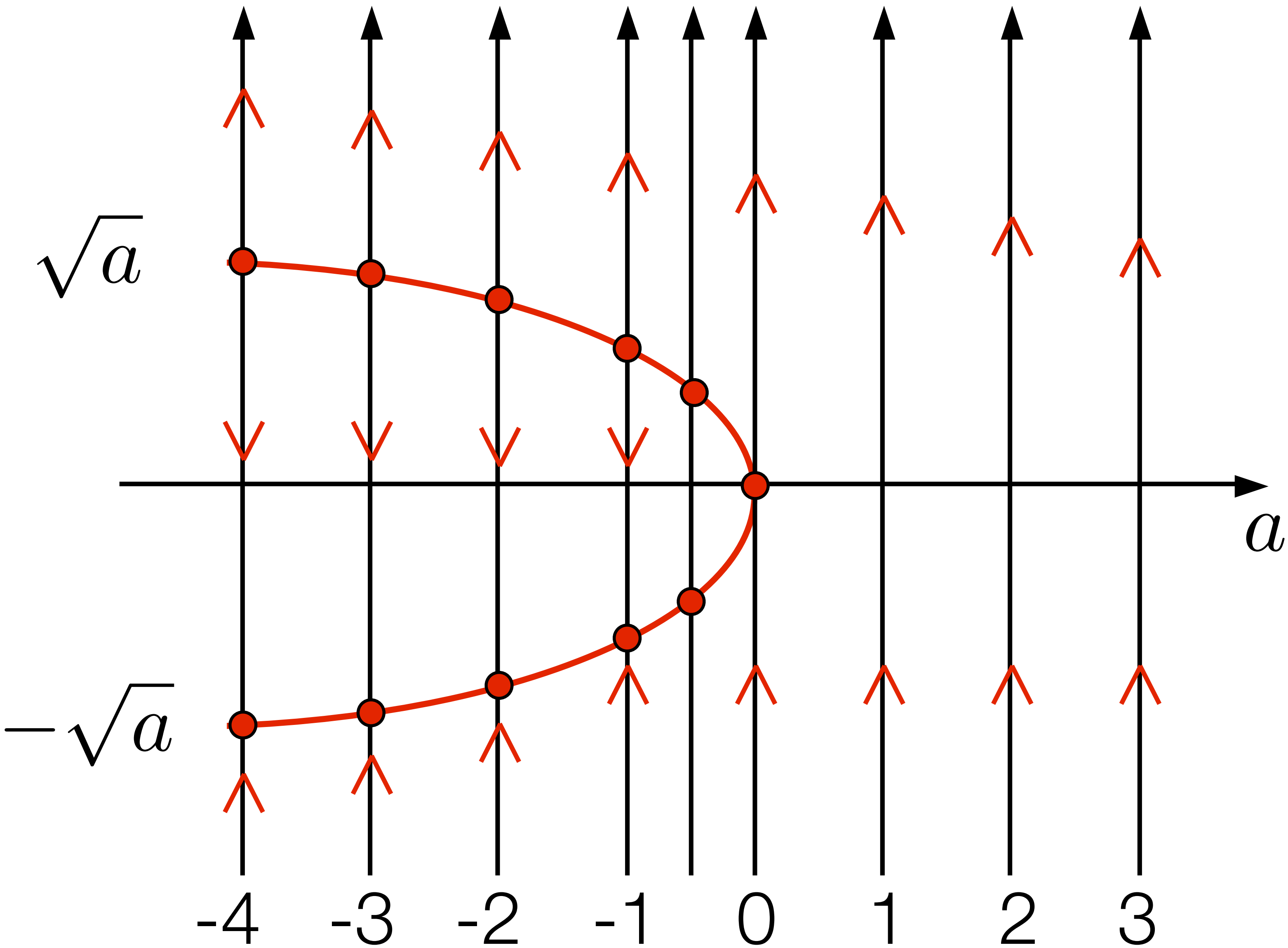
$$\frac{dy}{dt} = F_a(y) = y^2 + a$$

bifurcation at $a = 0$

$$F_a(y) = y^2 + a$$



Bifurcation Diagram



Linear 1st order ODEs

$$\boxed{\frac{dy}{dt} + g(t) \cdot y = r(t)}$$

Examples:

$$\frac{dy}{dt} + 2y = 0$$

$$\frac{dy}{dt} + 4y = \sin(t)$$

$$\frac{dy}{dt} + \frac{\tan(t)}{\sqrt{5}t} \cdot y = \cos^5(t^3)$$

Linear 1st order ODEs

$$\frac{dy}{dt} + g(t) \cdot y = r(t)$$

No nonlinear terms in y

e.g. $y^2, \sin(y), \frac{1}{y},$ etc.

Two cases:

Homogeneous: $\left(r(t) = 0 \right)$

$$\boxed{\frac{dy}{dt} + g(t) \cdot y = 0}$$

Separable: $$\frac{dy}{dt} = -g(t) \cdot y$$

$$\frac{1}{y} dy = -g(t) dt$$

$$\int \frac{1}{y} dy = -\int g(t) dt$$

Non-homogeneous: $\left(r(t) \neq 0\right)$

$$\frac{dy}{dt} + g(t) \cdot y = r(t)$$

Non separable

Need another solution technique

1. Find general solution for (H) equation: $y_{\mathrm{H}}(t)$

2. Find particular solution for (NH) equation: $y_{\mathrm{p}}(t)$

- General solution for (NH) equation:

$$y_{\mathrm{NH}}(t) = y_{\mathrm{H}}(t) + y_{\mathrm{p}}(t)$$

Step by step

- Find general solution for (H) equation: $y_{\mathrm{H}}(t)$

$$\boxed{\frac{dy}{dt} + g(t) \cdot y = 0} \quad \text{(H)}$$

- Guess a solution for (NH) equation: $y_{\mathrm{p}}(t)$

$$\boxed{\frac{dy}{dt} + g(t) \cdot y = r(t)} \quad \text{(NH)}$$

- General solution for (NH) equation:

$$y_{\mathrm{NH}}(t) = y_{\mathrm{H}}(t) + y_{\mathrm{p}}(t)$$

Art of guessing solutions

- Guess a solution for (NH) equation: $y_\mathrm{p}(t)$

$$\boxed{\frac{dy}{dt} + g(t)\cdot y = r(t)} \qquad \text{(NH)}$$

- Guess a function similar to $r(t)$

- Example:

$$r(t) = -2t \quad \text{guess} \quad y_\mathrm{p} = At$$

$$r(t) = 3e^{2t} \quad \text{guess} \quad y_\mathrm{p} = Ae^{2t} \qquad A \in \mathbb{R}$$

Plug into (NH) and find A

If a guess fails

- Example:

$$r(t) = 3e^{2t}$$

first guess: $y_{\mathrm{p}} = Ae^{2t}$

- if first guess fails.

second guess: $y_{\mathrm{p}} = A \cdot t \cdot e^{2t}$

- if second guess fails.

third guess: $y_{\mathrm{p}} = A \cdot t^2 \cdot e^{2t}$

Examples

$$\boxed{\frac{dy}{dt} + g(t) \cdot y = r(t)}$$

$$\frac{dy}{dt} + 4y = e^{-2t}$$

$$\frac{dy}{dt} + 4y = 2$$

$$\frac{dy}{dt} + y = 5e^{4t} \qquad y(0) = \pi$$

Examples

$$\frac{dy}{dt} + 2y = \sin(t)$$

guess $A\sin(t) + B\cos(t)$

$$\frac{dy}{dt} - y = \cos(4t)$$

guess $A\sin(4t) + B\cos(4t)$