

Day 9: July 12th

- **Chapter: 2.1 Modeling via Systems**
- Homework:
 - 1.1 Page 19 #19, 20.
 - 2.1 Page 164 #1-4, 9, 10, 15.
- **Chapter: 2.2 The Geometry of Systems**
- Homework:
 - Page 182 #1, 3, 9, 11-27 odd.

Part II: Systems of ODEs

- **Things in the real world are interconnected**
 - Populations:
 - #predators \longleftrightarrow #prey
 - Weather:
 - temperature \longleftrightarrow wind-speeds
 - Financial markets:
 - unemployment \longleftrightarrow spending
 - Neuroscience:
 - hippocampus \longleftrightarrow entorhinal cortex
 - **WARNING things can get complicated**

- **Chapter: 2.1 Modeling via Systems**

- Unlimited growth

$$\frac{dP}{dt} = kP$$

just depends
on the population

- Logistic population model

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N} \right)$$

- Interacting Populations:

Predator-Prey System:

- rabbits.
- foxes.

Predator-Prey Systems

- Assumptions
- Rabbits:
 - reproduce at a rate prop. to population if no foxes.
 - foxes eat rabbits at a rate prop. to # of rabbit/fox meetings.
- Foxes
 - if no rabbits, fox population decreases at a rate proportional to population
 - fox population increases at a rate prop. to # of rabbit/fox meetings.

Predator-Prey Systems

- Variables

$t = \text{time}$

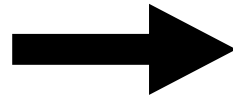
$R = R(t) = \# \text{ rabbits}$

$F = F(t) = \# \text{ foxes}$

- Describe change with time

$$\frac{dR}{dt} = ?$$

$$\frac{dF}{dt} = ?$$



$R(t)$

$F(t)$

Predator-Prey Systems

- Model Rabbits

- reproduce at a rate prop. to population if no foxes.

$$\frac{dR}{dt} = k \cdot R \quad \text{if only rabbits}$$

- foxes eat rabbits at a rate prop.

to # of rabbit/fox meetings: $\alpha \cdot R \cdot F$

$$\frac{dR}{dt} = k \cdot R - \alpha \cdot R \cdot F$$

Predator-Prey Systems

- Model Foxes

- if no rabbits, fox population decreases at a rate proportional to population

$$\frac{dF}{dt} = -l \cdot F \text{ if no rabbits}$$

- fox population increases at a rate prop. to # of rabbit/fox meetings

$$\frac{dF}{dt} = -l \cdot F + \beta \cdot R \cdot F$$

Predator-Prey Systems

- System of equations:

$$\frac{dR}{dt} = k \cdot R - \alpha \cdot R \cdot F$$

$$\frac{dF}{dt} = -l \cdot F + \beta \cdot R \cdot F$$

Two differential equations

- Non-linear.
- Must solve both at the same time.

Predator-Prey Systems

- More general case:

$$\frac{dR}{dt} = k \cdot R \cdot \left(1 - \frac{R}{N}\right) - \alpha \cdot R \cdot F$$

$$\frac{dF}{dt} = -l \cdot F + \beta \cdot R \cdot F$$

Two differential equations

- Non-linear.
- Must solve both at the same time.

What can we do?

- Geometry of systems
 - Section 2.2
- Look at a simpler case: linear systems
 - Chapters 3 and 4
- Return to non-linear systems.
 - linearize
 - develop new tools
 - Chapter 5

Predator-Prey Systems

- Special case:

$$\frac{dR}{dt} = R \cdot (1 - F) \quad \text{rabbits}$$

$$\frac{dF}{dt} = F \cdot (-1 + R) \quad \text{foxes}$$

- First: Equilibrium Points.

$$\frac{dR}{dt} = 0 \quad \text{and} \quad \frac{dF}{dt} = 0$$

Predator-Prey Systems

- Special case:

$$\frac{dR}{dt} = R \cdot (1 - F) \quad \text{rabbits}$$

$$\frac{dF}{dt} = F \cdot (-1 + R) \quad \text{foxes}$$

if no foxes: $F = 0$

$$\frac{dF}{dt} = 0$$

$$F(t) = 0$$

$$\frac{dR}{dt} = R$$

$$R(t) = c \cdot e^t$$

Predator-Prey Systems

- Special case:

$$\frac{dR}{dt} = R \cdot (1 - F) \quad \text{rabbits}$$

$$\frac{dF}{dt} = F \cdot (-1 + R) \quad \text{foxes}$$

if no rabbits: $R = 0$

$$\frac{dR}{dt} = 0$$

$$R(t) = 0$$

$$\frac{dF}{dt} = -F$$

$$F(t) = \bar{c} \cdot e^{-t}$$

Predator-Prey Systems

- Special case:

$$\frac{dR}{dt} = R \cdot (1 - F) \quad \text{rabbits}$$

$$\frac{dF}{dt} = F \cdot (-1 + R) \quad \text{foxes}$$

- Phase Plane: 2-D equivalent to phase line
 - solutions travel in phase plane.
 - PredatorPrey software.

Predator-Prey Systems

- Vector field: equivalent to slope field

- plot slope in phase plane (F vs R)

at (R, F) plot $V(R, F) = \left(\frac{dR}{dt}, \frac{dF}{dt} \right)$

- a solution curve is always tangent to V.

- solutions follow V.

- only for autonomous equations.

- Direction field = scaled vector field

- all vectors set to the same length.