**MA 471-671**

**Fall 2004**

**Final Exam**

**Name:** ______________________________

Do all problems and show all work.

**MA 471-671**
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1. True/False

a. ______ The function $F(x) = x^2 - 2$ has infinitely many eventually fixed points.

b. ______ The function $F(z) = z^2 - 1.705$ has a connected filled Julia set.

c. ______ The Sierpinski triangle has fractal dimension $\log 2 / \log 3$.

d. ______ The complex derivative of the function $F(z) = |z|^2$ is $2|z|$.

e. ______ The Schwarzian derivative of $F(x) = e^x$ is constant.

2. Quickies. Answers only – no partial credit.

a. The functions $F(x) = x^2 + c$ has a _______________ bifurcation at $c = -5/4$.

b. Describe the behavior of all orbits of the function

$$F(x) = \begin{cases} -x & \text{if } x \geq 1 \\ -2 + x & \text{if } x < 1 \end{cases}$$

c. Draw a sketch of the 3/7-bulb hanging off the main cardioid of the Mandelbrot set (with particular attention to the main antenna).

d. The doubling map of the unit circle $F(\theta) = 2\theta$ has a cycle of prime period 4 at: (give the "angle" (mod 1) of one such point).

e. For which values of $c$ does the complex quadratic function $F(z) = z^2 + c$ have two distinct fixed points?

f. List the Sarkovskii ordering of periods for continuous functions on the real line.

3. Definitions. Give the precise definitions of each of the following.

a. Filled Julia set.

b. The function $F$ is chaotic on a set $S$.

c. $H$ is a conjugacy between $F$ and $G$.

d. $Q$ is a dense subset of the set $T$.

4. Describe in an essay complete with pictures all aspects of the saddle node bifurcation (real and complex) that occurs for the function $F(z) = z^2 + c$ when $c = 1/4$.

Continue your answer to question 4 here.

5. In an essay complete with pictures, discuss the Fundamental Dichotomy for the family of complex functions $F(z) = z^2 + c$. Be sure to include the relationship between this dichotomy and the Mandelbrot set.

Continue your answer to question 5 here.

6. Consider the shift map $\sigma$ on the sequence space $\Sigma_2$.

a. Is the shift map continuous? If so, prove it. If not, explain why.

b. Is the shift map onto? If so, prove it. If not, explain why.

c. Is the shift map one-to-one? If so, prove it. If not, explain why.

Continue your answer to problem 6 here.

7. Is the set of all sequences that have exactly three 0's in the sequence dense in the sequence space $\Sigma_2$? If so, prove it. If not, explain why not.

7. Continued. Continue your answer to question 7 on this page, or use it for scrap.