MA 471-671
Fall 2005
Final Exam
Name: _____

Do all problems and show all work.

MA 471-671
Fall 1997
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1. True/False

a. _____ The function $F(x) = x^2 - 2$ has infinitely many eventually fixed points.

b. _____ The function $F(x) = x^2$ is conjugate to $G(x) = 4x(1-x)$.

c. _____ The subset consisting of the endpoints of the middle thirds Cantor set is dense in the Cantor set.

d. _____ The complex derivative of the function $F(z) = |z|^2$ is $2|z|$.

e. _____ The Schwarzian derivative of $F(x) = \tan x$ is constant.

2. Quickies. Answers only – no partial credit.

a. The fractal dimension of the Sierpinski tetrahedron is:

b. Describe the behavior of all orbits of the function

$$F(x) = \begin{cases} -x & \text{if } x \geq 1 \\ -1 + x & \text{if } x < 1 \end{cases}$$

c. Draw a picture of the 3/7-bulb hanging off the main cardioid of the Mandelbrot set (with particular attention to the main antenna).

d. The doubling map of the unit circle $F(\theta) = 2\theta$ has a cycle of prime period 5 at: (give one such point)

e. For which values of c does the complex quadratic function $F(z) = z^2 + c$ have two distinct fixed points?

f. List the Sarkovskii ordering of periods for continuous functions on the real line.

3. Definitions. Give the precise definitions of each of the following.

a. Filled Julia set

b. The function F is chaotic on a set S.

c. H is a conjugacy between F and G.

d. Q is a dense subset of the set T

4. Describe in an essay complete with pictures all aspects of the saddle node bifurcation (real and complex) that occurs for the function $F(z) = z^2 + c$ when $c = 1/4$.

Continue your answer here.

5. Consider the following point in the sequence space Σ: $(0\,1\,00\,1\,000\,1\,0000\,1\ldots)$. Is the orbit of this point under the shift map dense in Σ? If so, prove it. If not, give a complete reason.

6. Describe the behavior of orbits for the complex linear function $F(z) = e^{2\pi i\theta}z$ for all possible values of θ.

7. The Mandelbrot set. Write an essay describing the Mandelbrot set. In your essay you should discuss the arrangement of the bulbs around the main cardioid as well as what the Mandelbrot set means. You should include in your essay a specific formula for the boundary of the main cardioid.

7. Continued. Continue your answer to question 6 on this page, or use it for scrap.