Boston University

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Homework Assignment 1

Exercise 1

Let ξ and ζ be two orthogonal functions in $L^2([0,T])$. Compute

1. $I_1(\xi)I_2\left(\zeta^{\otimes 2}\right)$;
2. $I_3\left(\widetilde{\xi \otimes \zeta^{\otimes 2}}\right)$;
3. $D_t I_3\left(\widetilde{\xi \otimes \zeta^{\otimes 2}}\right)$;
4. $D_t\left(I_1(\xi)I_2\left(\zeta^{\otimes 2}\right)\right)$;
5. $D_t W(T)$;
6. $D_t \int_0^T s^2 dW(s)$;
7. $D_t e^{\int_0^T g(s)dW(s)}$, $g \in L^2([0,T])$.

Exercise 2

Suppose that $F = \sum_{n=0}^{\infty} I_n(f_n)$, $f_n \in H^{\odot n}$, is in $\mathbb{D}^{\infty,2} = \bigcap_k \mathbb{D}^{k,2}$. Prove that then, $f_n = \frac{1}{n!}\mathbb{E}(D^n F)$ for every $n \geq 0$.

Exercise 3

Let F_1 and F_2 be in $\mathbb{D}^{1,2}$ such that F_1 and $\|DF_1\|_H$ are bounded. Show that $F_1F_2 \in \mathbb{D}^{1,2}$ and $D(F_1F_2) = F_1DF_2 + F_2DF_1$.

Exercise 4

Suppose $W = \{W(t) \colon 0 \leq t \leq 1\}$ is a Brownian motion. Show that $M = \sup_{0\leq t\leq 1} W(t) \in \mathbb{D}^{1,2}$ and $D_t M = \mathbb{1}_{[0,T]}(t)$ where T is the a.s unique point where W attains its maximum.