

BOSTON UNIVERSITY

MALLIAVIN CALCULUS, FALL 2015

Homework Assignment 2

EXERCISE 1

Compute the following Skorohod integrals :

1. $\int_0^T W(t) \delta W(t)$;
2. $\int_0^T \int_0^T g(s) dW(s) \delta W(t)$, $g \in L^2([0, T])$;
3. $\int_0^T e^{W(T)} \delta W(t)$.

EXERCISE 2

Compute $D_t F$ where $F = \int_0^T W(t) \delta W(t)$, $t_0 \in [0, T]$.

EXERCISE 3

Compute $D_t F$ where $F = e^G$ and $G = \int_0^T g(s) dW(s)$, $g \in L^2([0, T])$. Verify that it holds that $D_t e^G = e^G D_t G$.

EXERCISE 4

Prove that for $F \in \mathbb{D}^{1,p}$, $p \geq 2$ and $T = [0, 1]$,

$$\mathbb{E}(|F|^p) \leq C_p \left[\mathbb{E}(|F|^p) + \mathbb{E} \left(\int_0^1 |D_t F|^p dt \right) \right].$$

EXERCISE 5

Find an explicit representation (without any conditional expectation) of $M = \sup_{t \in [0, 1]} W(t)$, where W denotes a Brownian motion on $[0, 1]$.
