MA 127 A1 **Final Exam**

December 19, 1998

This exam is a closed book, no notes, no "crib sheets" exam. Calculators are permitted, though on some problems I have specified that they should not be used. There are eleven problems on this exam – don't overlook the one on the back of the page. The number of points that each problem is worth is printed next to the problem. Good luck!

1. Evaluate the derivatives of the following functions. (5 points each)

$$(a) \quad y(x) = \frac{x}{1+e^x} \ , \quad (b) \quad f(t) = \sin(\ln(t)) \ .$$

2. On what intervals is the function $f(x) = 2\sin x + \sin^2 x$ increasing? Please show your work – no credit for answers obtained from the calculator. (7 points)

3. Evaluate the following integrals. You must show your work – calculator answers are not acceptable. (5 points each)

$$(a) \quad \int \frac{e^x}{1+e^x} dx \ , \quad (b) \quad \int_0^{\pi/4} x \sec^2 x dx \ .$$

4. Determine the limit

$$\lim_{x \to 0} (1-2x)^{1/x} \ .$$

Please show your work – no credit for answers obtained from the calculator. (7 points)

5. Determine whether the following infinite series converge or diverge. Explain which method you use to determine convergence or divergence. (5 points each)

$$(a) \sum_{n=3}^{\infty} \frac{\ln n}{n} \ , \quad (b) \sum_{n=1}^{\infty} n e^{-n} \ .$$

6. Compute the volume of the solid obtained by revolving the region bounded by $y = x^3$, $y = 8$, and $x = 0$ *about the y-axis.* (10 points)

7. Determine the radius of convergence of the power series (8 points)

$$\sum_{n=0}^{\infty} n(x-1)^n \ .$$

8. Find the third order Taylor polynomial, $T_3(x)$, for $f(x) = \tan^{-1} x$, with center $a = 0$. Please show your work. (8 points)

9. A rectangular storage container with an open top is to have a volume of 10 m^3. The base of the container is a square. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container. (10 points)

10. A paper cup has the shape of a cone with height 10 cm and radius 3 cm at the top. If water is poured into the cup at a rate of 2 cm^3/sec, how fast is the water level rising when the water is 5 cm deep? (*Hint:* The volume of a cone with height h and radius at the top r is $V = \frac{1}{3}\pi r^2 h$.)(10 points)

11. Explain carefully how to define the integral $\int_a^b f(x)dx$ as a limit of Riemann sums. (10 points)