MA 127 A1 **Midterm Exam #1**
October 2, 1998

This exam is a closed book, no notes, no "crib sheets" exam. Calculators are permitted. There are nine problems on this exam – don't overlook those on the back of the page. All problems are worth 11 points each, except for problem #1 which is worth 12 points – 6 for part (a) and 6 for part (b) Good luck!

1. Determine whether or not the following limits exist. If they exist, compute them. If not, explain why not:

$$(a)\ \lim_{x\to -3}\frac{x^2-x+12}{x+3},\ (b)\ \lim_{t\to 0}\frac{\tan(4x)}{x}\ .$$

2. Evaluate the derivative of $f(x) = xe^{-1/x}$.

3. Let $f(x) = \log_2(x^2+2x+8)$. Find the equation of the tangent line to the graph of f at $x = 2$.

4. Let $h(x) = f(g(x))$, with f and g both differentiable. Suppose that we are given the following values for f, f', g and g':

$$f(1) = -1, f'(1) = 2, f(2) = 0, f'(2) = -2,$$
$$g(1) = 2, g'(1) = 3, g(2) = -2, g'(2) = 1.$$

Use the chain rule to find the value of $h'(1)$.

5. Use implicit differentiation to find the equation of the tangent line to the curve defined by

$$x^2 = y^3(2+y)$$

at the point $(\sqrt{3}, 1)$.

5. Sketch the graph of a function satisfying the following conditions:

- $f'(-1) = 0$, $f'(1)$ does not exist.
- $f'(x) < 0$, if $|x| < 1$, $f'(x) > 0$ if $|x| > 1$.
- $f(-1) = 4$, $f(1) = 0$.
- $f''(x) > 0$, if $x \neq 1$.

Note: There are many functions f which satisfy these conditions. It suffices to find one of them.

7. Let $C(t)$ be the cost of living as a function of time. Explain what the sentence "The cost of living continues to increase but at a slower rate" means in terms of the function $C(t)$ and its first and second derivative.

8. The US population from 1920 to 1980 can be approximated reasonably well by the function

$$P(t) = 2325.67t^3 - (1.306 \times 10^7)t^2 + (2.45 \times 10^{10})t - (1.53 \times 10^{13})$$

Here, t is the time, in years, *i.e.* $P(1960)$ is the population in 1960. Use the linear approximation to $P(t)$ at $t = 1980$ to estimate $P(1981)$ and $P(1984)$. Which approximation do you expect to be most accurate? Why?

9. Company X sells widgets. At the moment it has $12,156$ customers each of whom buys 5.67 widgets per month, on average. It is losing customers at the rate of 617 per month, but the rate at which current customers are buying widgets is increasing at the rate of 1.12 widgets per month per month. Use the product rule to compute the total rate of change in the company's monthly widget sales. Explain what each term in the expression given by the product rule means in terms of the company's sales.