MA 127 A1 **Midterm Exam #3**
November 23, 1998

This exam is a closed book, no notes, no "crib sheets" exam. Calculators are permitted, though on some problems I have specified that they should not be used. There are seven problems on this exam – don't overlook the one on the back of the page. The number of points that each problem is worth is printed next to the problem. Good luck!

1. Evaluate the indefinite integral: (14 points)

$$\int x \cos x dx \ .$$

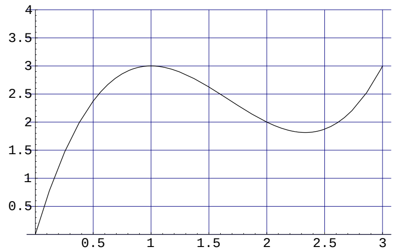
(Please show your work – no credit for "calculator" answers.)

2. Evaluate the definite integral (14 points)

$$\int_0^{\pi/2} (\sin x) \cos(\cos x) dx \ .$$

(Please show your work – no credit for "calculator" answers.)

3. Use the trapezoid rule with $n = 3$ to approximate $\int_0^3 f(x)dx$ for $f(x)$ the function whose graph is given below. (14 points)



4. Find the area of the region enclosed by $y = x^2$ and $y = 2/(x^2 + 1)$. (14 points)

5. Find the volume of the solid one obtains when the region enclosed by $y = \cos x$, $y = \sin x$, $x = 0$, and $x = \pi/4$ is revolved about the x-axis. (**Hint:** $\cos^2 x - \sin^2 x = \cos(2x)$.)
(14 points)

6. Note that the function $f(x) = \frac{1}{x\sqrt{\ln x}}$ has a vertical asymptote at $x = 1$, and $f(1)$ is undefined.

- Explain how to define the improper integral (7 points)

$$\int_1^3 \frac{dx}{x\sqrt{\ln x}} \ .$$

- Does this integral converge or diverge? If it converges, compute its value. If it diverges, explain why. (7 points)

7. The following questions all deal with work. (4 points for each part.)

(a) If one exerts a force $f(x)$, to move an object from a to b, what is the formula for the work done on that object?

(b) The force exerted by an ideal spring is $F = -kx$, where k is a constant, and x is the amount the spring is stretched beyond its natural length. Find a formula for the amount of work done to stretch a spring an amount L from its natural length.

(c) Suppose that 1.56 J of work is needed to stretch a spring from its natural length of .1 meters, to a length of .15 meters. What is the value of the constant k in the formula for the force exerted by the spring?

(d) Consider the spring discussed in part (c). How much work is necessary to stretch it from .15 meters to .2 meters?