MATH 562
Homework 2
Due Tuesday February 19

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"La Physique ne nous donne pas seulement l'occasion de résoudre des problèmes ..., elle nous fait presentir la solution."

Henri Poincaré

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Page 217: Problems 1, 3, 5 (feel free to use a computer rather than doing this problem "by hand").

Problem 4. Write a scheme to solve the transport equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \ ,$$

using forward differences for both x and t. Analyze the stability of your scheme – how does it depend on a? Does the sign of a matter? (You solved this equation in section 1.2 of Strauss' book. Can you relate the exact solution to your numerical stability condition in the way we did with the stability condition for the wave equation?

Problem 5: Consider Burger's equation

$$u_t + uu_x = 0$$

with initial conditions,

$$u(x,0) = 1 - |x| \ ,$$

for $|x| \leq 1$ and $u(x,0) = 0$ otherwise. Suppose that $-3 \leq x \leq 3$ and that we have zero Dirichlet boundary conditions. Try to solve this equation numerically using both a standard finite difference method (your choice). Use a fine mesh (at least 50 points - 100 would be better) and observe very carefully what happens as $t \to 1$. Plot the numerically computed solution for several values of t close to 1. Can you continue the solution past $t = 1$? (Note that Burger's equation is a non-linear partial differential equation. It's behavior is very different from that of a linear equation even though the only difference between it and the transport equation considered in Problem 4 is that the "a" in Problem 4 is replaced by a "u" here.)