MATH 574
Homework 3
Due Thursday March 8

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"An eloquent mathematician must, from the nature of things, remain ever as rare a phenomenon as a talking fish."

J.J. Sylvester

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The first two problems on singular perturbation theory come from the book, **Mathematics Applied to Deterministic Problems in the Natural Sciences** by Lin and Segal.

1. Consider the equation

$$\epsilon y'' + (1+x)y' + y = 0 \ , \ y(0) = 0 \ , \quad y(1) = 1 \ .$$

Assume that ϵ is small and positive and that the solution has a boundary layer at the origin.

(a) Find the leading order outer approximation to the solution.

(b) Find the leading order inner approximation to the solution.

(c) Use the method of matching to determine any unknown constants and then find the uniform approximation.

2. A "small" mass m hangs from a weightless spring with internal damping proportional to speed. A vertical impulse I is imparted to teh mass by striking it with a hammer. Initial conditions on the vertical deflection y^* at time $t^* = 0$ can be taken to be

$$y^*(0) = 0 \ , \quad m\frac{dy^*}{dt^*}(0) = I \ .$$

The governing equation is

$$m\left(\frac{d^2y^*}{dt^{*2}}\right) + \mu\frac{dy^*}{dt^*} + ky^* = 0 \ ,$$

where μ and k are the damping and spring constants.

(a) Show that a certain choice of dimensionless variables reduces the problem to

$$\epsilon y'' + y' + y = 0 \ , \quad y(0) = 0, \quad \epsilon y'(0) = 1 \ .$$

(b) Find the outer approximation. Do not impose any initial conditions on the outer approximation.

(c) Find the inner approximation. Impose both initial conditions on the inner solution.

(d) Use the matching method to determine the remaining constant in the outer solution.

(e) Find the exact solution of the problem and compare it with the composite approximation for $\epsilon = 0.1$ and $\epsilon = 0.03$.

From the textbook please also do:

Problem 9.3.4 (page 361).

Problem 12.2.5 (page 524).

Problem 12.2.8 part (a) (page 525).