CAS MA 575 – Linear Models.

Boston University, Fall 2013

HOMEWORK 2

1. For any set of realizations, $\{x_1, \ldots, x_n\}$, from a random variable X, verify the following equality:
$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i - \bar{x})x_i,$$
where $\bar{x} := \frac{1}{n}\sum_{i=1}^{n} x_i$.

2. In appendix A.3 of your textbook, verify that the partial derivatives of $\mathrm{RSS}(\beta_0, \beta_1)$ with respect to β_0 and β_1 are respectively given by
$$\begin{aligned} \frac{\partial}{\partial\beta_0}\mathrm{RSS}(\beta_0,\beta_1) &= -2\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i), \\ \frac{\partial}{\partial\beta_1}\mathrm{RSS}(\beta_0,\beta_1) &= -2\sum_{i=1}^{n} x_i(y_i - \beta_0 - \beta_1 x_i). \end{aligned} \tag{1}$$

3. Solve the two equations in (1) with respect to β_0 and β_1, in order to obtain the estimates,
$$\widehat{\beta}_0 = \bar{y} - \widehat{\beta}_1\bar{x}, \qquad \text{and} \qquad \widehat{\beta}_1 = \frac{\mathrm{SXY}}{\mathrm{SXX}},$$
respectively; where $\mathrm{SXY} := \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$ and $\mathrm{SXX} := \sum_{i=1}^{n}(x_i - \bar{x})^2$.

4. Problem 1.2 from your textbook.

5. Problem 2.1 from your textbook. (This involves material that will be covered next week.)