Chapter 2

Practical Matters

What a waste it is to lose one's mind. Or not to have a mind as being truly wasteful. How true that is.

Dan Quayle, former Vice President of the U.S.

In God we trust; all others bring data.

A Statistician

All innovations succeed in the hands of the innovator, and none succeed in other hands.

David S. Moore

You may as well put the sidewalk where the students walk, because they'll walk there anyway.

William James Lewis

The pleasure of learning, and knowing, though not the keener, is yet the least perishable of pleasures; the least subject to external things, and the play of chance, and the wear of time. And as a prudent man puts money by to serve as a provision for the material wants of his old age, so too he needs to lay up against the end of his days provision for the intellect.

A. E. Houseman

People who become legends in their own time usually have very little time left.

John D. MacDonald

The most irresponsible thing that a civilized adult can do is to stand up in a crowded room and shout "Proof!"

James Carlson

Everybody will be famous for fifteen minutes.

Andy Warhol

An expert is just some guy from out of town.

Mark Twain

2.0 Chapter Overview

Like many activities in life, fine teaching is composed of many technical components. When everything works properly, then the whole is considerably greater than the sum of its parts. However, if some of the crucial parts are rusty or, worse, non-functional, then the whole will creak and drag and not do a good job of it.

The novice instructor should probably read every section in this chapter. The more experienced instructor may wish to pick out particular sections for concentrated effort.
2.1 Voice

There is nothing more stultifying than a lecture in a reasonably large classroom on a hot day delivered by an oblivious professor mumbling to himself at the front of the room. We are not all actors or comedians or even great public speakers. But we are teachers, and we must convey a body of material. We must capture the class’s attention. We must fill the room.

If you are unlucky, you may be assigned to teach in a classroom that works against you. Perhaps visibility is poor for the students in the back, or the acoustics are bad, or the blackboard is substandard. If you are burdened with such a teaching environment, try your best to get it changed. If you cannot, then think hard about how to get the best out of this classroom. If the blackboard is unusable, then consider lecturing with an overhead projector. If the acoustics are bad, then consider using a microphone. If visibility is poor, then think about changing the seating arrangement. No matter what the liabilities, you must take charge.

Your voice is one of your primary tools. And you must use that tool in part to control the environment in your classroom. The most important presence in the room is not the blackboard, nor the desk, nor the text. It is you. You want the students’ attention focused on you.

I am not saying that you must lose your dignity, or act silly, or show off. You must learn to use your voice and your eyes and your body and your presence as a tool. If you are going to say something important, then make a meaningful pause beforehand. Say that it is important. Repeat the point. Write it down. Give an example. Repeat it again.

You can gain the attention of a large group by lowering your voice. Or by raising it. Or by pausing. One thing is certain: You will not gain the audience’s attention by rolling along in an uninfected monotone. Again, I am not suggesting that you undergo a personality change in order to be a sound teacher. What I am suggesting is that you find ways to talk to them as a person interacting with people, rather than as an ill-at-ease, out-to-lunch egghead.

At a well-known university (of good quality!) in southern California they once tried bringing in actors from Hollywood to help professors spice up their delivery. One instructor of an Abnormal Psychology course was advised to use the line, “I never teach about any mental illness until I try it out myself.” Such pandering is inappropriate, offensive, and childish. Can you imagine yourself using such patter? Who would want to?

What I am suggesting here is that you take just a little time and contemplate your lecture/classroom style. A lecture or class should be a controlled conversation between you and your audience. It is a trifle one-sided, of course. But there must be cerebral interaction between the teacher and the students. That means that you, the instructor, must grab and maintain the attention of the class. Your behavior in front of the group is a primary tool for keeping the lines of communication open.

When you are talking about a subject that you perceive to be trivial, and when you are nervous, you tend to talk too fast. Novice instructors find themselves barreling through their lectures. You must resist this tendency. If you are
2.1. VOICE

really new at the business of teaching, then practice your lectures. Get a friend to listen. In calculus, a fifty minute lecture with three or four good examples and some intermediate explanatory material is probably just about right (I’m thinking here of a lecture on max-min problems, for example). Try to make each class consist of about that much material, and make it fill the hour. If you finish early, that is fine (but it may mean that you talked too fast). You can quit early for that day, or do an extra example, or use the extra time to answer questions.

Don’t give the students the impression that you are in a rush. It puts them off, and reflects a bad attitude toward the teaching process. If on Wednesday you plan to explain the chain rule, then do just that. If the chosen topic does not fill the hour, then do an extra example or field questions. Do not race on to the next topic. One idea per class, at the lower-division level, is about right. (Of course if you are teaching a multi-section class at a big university, then it is important to keep pace with the other instructors. This is yet another reason for keeping careful track of your use of time. See also Section 1.9.)

It is something of an oversimplification, but still true, that a portion of the teacher’s role is as a cheerleader. You are, by example, trying to persuade the students that this ostensibly difficult material is doable. Part of the secret to success in this process is to have a controlled, relaxed voice, to appear to be at ease, and to be organized. Don’t let a small error fluster you. Make it seem as though such a slip can happen to anyone, and that fixing it is akin to tying your shoelaces or pulling up your socks.

But, as with all advice in this book, you must temper the thoughts in the last paragraph with a dose of realism. If you make the material look easy, then students will infer that it is easy. The psychological processes at play here are not completely straightforward. Nobody would be foolish enough to go to an Isaac Stern concert and come away with the impression that playing the violin is trivial. Yet students attend my calculus classes, watch me solve problems, conclude that the material is easy and that they have it down cold. They decide that in fact they don’t need to do any homework problems or read the book, and then they flunk the midterm.

These are the same students who come to me after the exam and say, “I understand all the ideas. The material is absolutely clear when you talk about it in class. But I couldn’t do the problems on the exam.” How many times have you heard such a statement from your students? I like to tease my students by reminding them that this is like saying, “I really understand how to swim, but every time I get in the water I drown.”

On the one hand, you don’t want to make straightforward material look difficult. After 300 years, we’ve got calculus sewn up. There is no topic in the course that is intrinsically difficult. We merely need to train our students to do it. So do make each technique look straightforward. But remind the students that they themselves need to practice. Do this by telling them so, by giving quizzes, by varying the examples and introducing little surprises. Ask the class questions to make the students turn the ideas over in their own minds. Use your voice to encourage, to wheedle, to cajole, to question, to stimulate.

Mathematics instructors in general, whether they are “reformers” or “traditionalists” or “high techers” or “plug-and-chuggers”, agree that each student
must take each idea in the course and rebuild it in his own mind. This is nothing new. Go read Beth and Piaget [BPA]—they discuss this notion in detail. An awareness of this concept will help you to shape your teaching methods. If the students cannot understand what you are talking about, then it is unlikely that they will take the ideas home and think about them. If the students watch you state and prove a lot of abstruse theorems, and in the process become terminally depressed, then it is unlikely that they will take the ideas home and analyze them and internalize them. If the students watch you flounder around, unable to complete an example coherently or explain a concept neatly, then it is unlikely that they will take the ideas home and rebuild them in their own minds.

If instead you kindle the students’ curiosity, plant in them a desire to learn, show them something they have never seen before and make them realize that it is something they have never seen before—and certainly never understood before—then there is a real likelihood that they will leave class turning the new thoughts over in their minds, talk among themselves about it, ask questions, and come back to you with their own ideas. That is teaching.

Even if you know how to use your voice well with a small audience, and to capture their attention and get them excited about learning, there are special problems with the large classes that are used in the teaching of calculus (for instance) at many universities. Refer to Section 2.14 for more on this matter.

2.2 Eye Contact

We all know certain people who invariably emerge as the leader of any group conversation. Such people seem to sparkle with wit, erudition, and presence. They usually pick the topic and they usually aim the discussion. They have a sense of humor, and they are intelligent. What is their secret?

It is partly a matter of attention and awareness. The sort of person I am describing has an inborn curiosity; he is aware of you, and interested in you, and genuinely eager to learn about your opinions and experiences and interests. When you ask yourself what makes another person interesting, an honest answer would have to entail that such a person is outer directed, and cares about others.¹

This is obviously a talent that is partly inborn and partly cultivated. Some of the trick is to show genuine interest in what other people have to say before bounding ahead with what you have to say. Another part is to talk about subjects, and to tell anecdotes, that you know will interest other people. Being charming and witty helps too, but in this section I want to concentrate on more mechanical features of repartee.

Many of the devices that make for an engaging conversationalist also make for an engaging teacher. A review of the last paragraphs, and of the rest of this book, will bear out this assertion. In this section I will discuss the importance of eye contact.

Telling a good joke while staring at the floor with your thumb in your ear will not have the same effect as telling the joke while looking at your listener,

¹A boring person is one who talks about himself. An interesting person is one who talks about you.
engaging his attention, and reacting to the listener while the listener is reacting to you. A good joke teller has his audience starting to chuckle halfway through the joke and just dying for the punch line. Getting a good laugh is then a foregone conclusion.

Giving a good lecture or class is serious business, and is not the same as telling a joke. But many of the moves are the same. If you want to hold your audience's attention then you must look at your audience. You must engage not one person but all. You must learn to use your body as a tool. Step forward and back. Force the eyes of the audience to follow you. A good lecturer speaks to individuals in the audience, to grouplets in the audience, and to the whole audience. Like a movie camera, you must zoom in and zoom out to get the effects that you wish to achieve. A ninety minute movie filmed at the same constant focal length would be dreadfully boring. Ditto for a lecture.

Some people are very shy about establishing eye contact. It is a device that you must consciously cultivate. The end result is worth it. The teacher who can establish eye contact is also the teacher who is confident, who is well prepared, and who conducts a good class.

2.3 Blackboard Technique

Write neatly. Write either in very plain longhand or print. Be sure that your handwriting is large enough. Be sure that it is dark enough. Endeavor to write straight across the blackboard in a horizontal line. Proceed in a linear fashion. Don't have a lot of insertions, arrows, and diagonally written asides.

Don't put too much material on each board. The ideas stand out more vividly if they are not hemmed in by a lot of adjacent material. In particular, it is difficult for students to pay attention when the teacher fills the board with long line after long line of print. An excellent guitarist once said that the silences in his music were at least as important as the notes. When you are laying material out on a blackboard, the same can be said of the blank spaces.

Some people find it useful to divide the blackboard into boxes. This practice makes it easier for the lecturer to organize what he is writing, and also makes it much, much easier for the students in the audience to organize the material in their minds and in their notes.

Label your equations so that you can refer to them verbally. Draw sketches neatly. Use horizontal and vertical lines to set off related bodies of material.

You can control your output more accurately by keeping the length of each line short. Think of the blackboard as being divided into several boxes and write your lecture by putting one idea in each box. To repeat: If necessary, actually divide the blackboard into boxes.

If the classroom has sliding blackboards, think ahead about how to use them so that the most (and most recent) material is visible at one time. For those combinatorial theorists among you, or those experts on the game of NIM, this should be fun.

If you are right-handed, consider writing first on the right-hand blackboard and then working left. The reason? That way you are never standing in front of
what you’ve written. Good teaching consists in large part of a lot of little details like this. You shouldn’t be pathological about these details, but if you are aware that they are there then you will pick up on them.

Every now and then during your presentation, you should stand aside and pause. Don’t say anything. This gives you an opportunity to collect your thoughts and catch your breath. You can verify the accuracy of what you have written. It gives the students an opportunity to catch up and to ponder what they’ve heard. They may decide to formulate questions. If instead you are barging ahead at full speed for the entire hour, then students never have a moment to think about what they are hearing. They cannot interact with you because you are not interacting with them.

Try to think ahead. Material that needs to be kept—and not erased—should be written (probably in a box) on a blackboard to the far left or far right where it is out of the way but can be referred to easily. You may wish to reserve another box on the blackboard for asides or remarks. Some instructors put material that needs to be seen for the entire hour on an overhead slide. This frees up all the blackboards, and keeps those important equations or definitions front and center.

These ideas are another facet of the precept that you know the material cold so that you can concentrate on your delivery. Just as an actor knows his lines cold so that he can make bold entrances and exits, and not trip over his feet, so you must be able to focus a significant portion of your brain on the conveying of the information.

If your lesson will involve one or more difficult figures then practice them on a sheet of paper in advance. Remember that you are a mathematical role model for the students. If you make it appear that it is difficult for you to draw a hyperboloid of one sheet, then how are the students supposed to be able to do it? Of course you can prepare the figure ahead of time on an overhead slide (or even photocopy it straight out of a book, or straight from Mathematica or Maple, onto a transparency). This solves the problem of having a nice figure to show the students. It does not solve the problem of showing the students how to draw the figure. As a result, it puts a barrier between them and the ultimate goal of learning how to read the graph. If necessary, consult a colleague who is artistically adept for tips on how to draw difficult figures.

You will find students quite resistive to learning to graph—especially in three dimensions. I learned a useful teaching device when last I taught multi-variable calculus, and it became clear to me when I was showing my students how to graph. That device is persistence. I made it clear to them that anything that gave them a pain in the neck was going to appear repeatedly in subsequent work. For example, almost all of them did rather poorly on the graphing portion of the first midterm. So I gave them several followup quizzes to help them hone their graphing skills. After each quiz they all gave me their “Is that finally the end of graphing?” look. But after examining their work I said, “Nope; not good enough yet.” And on we went. Over and over again I graphed functions in three dimensions. I went through every step. And I did it at the blackboard, just as I expected them to do it with pencil and paper. Graphing appeared again on the final. And I told them that it would so appear. In fact I told them that the best way to study for the final was to find everything on the first two midterms
that they hated, and that this material would certainly be on the final. They believed me, and it worked.

If you cannot organize the steps of a maximum-minimum problem, then can you really expect the students to do so? In the best of all possible worlds, the students' work is but a pale shadow of your own. So your work should be the platonic ideal. Sometimes, in presenting an example or solving a problem at the blackboard, you may inadvertently gloss from one step to another. Or you might make a straightforward presentation look like a bag of tricks. This practice is very confusing for students, especially the ones who lack confidence. By organizing the solution in a step-by-step format you can avoid these slips.

After you have filled a board, it should be neat enough and clear enough that you could snap a Polaroid® snapshot and read the presentation from the Polaroid®. In particular, you should not lecture by writing a few words, erasing those, and then writing some more words on top of the erased old words. Students cannot follow such a presentation. I cannot emphasize this point too strongly: Write from left to right and from top to bottom. Do not erase. When the first box is filled, proceed to the second. Do not erase. Only when all blackboards are full should you go back and begin erasing. Students must be given time to stare at what they've just seen as well as what is currently being written. Keep as much material as possible visible at all times.

BUT: When it is time to erase, be sure to erase thoroughly. It is well worth spending a few extra moments being sure that the blackboard is sparkling clean before you begin a new block of material. For if you endeavor to write over a sloppily erased blackboard then your writing will be obscure at best. This is really a psychological issue. Of course the students can squint and strain and figure out what you are writing (even if it is a virtually unreadable palimpsest), but it bums them out to have to do so. Try to make their job as easy as possible.

Do not stand in front of what you are writing. Either stretch out your arm and write to the side or step aside frequently. Read aloud to the class as you write. Make the mathematics happen before their eyes and be sure that they can see everything. Every once in a while, pause and step aside to catch your breath and to let them catch up.

Here is a common error that is made even by the most seasoned professionals. Imagine that you do an example that begins with the phrase "Find the local maxima and minima of the function ..." And so forth. Say that you've worked the example. Now suppose that the next example begins with the same phrase. It is a dreadful mistake to erase all but that first phrase and begin the new example on the fly, as it were.

Why is this a mistake?—it seems perfectly logical. But the students are taking notes! How can they keep up when you pull a stunt like this? Slow yourself down. Write the words again. If a student gets two sentences behind then he may as well be two paragraphs behind. Give frequent respites for catch up.

And now a coda: How much of what you are saying should you write? In my experience, the answer is "As much as possible." When you are transmitting sophisticated technical ideas verbally, students have trouble keeping up. Many
of them are not native English speakers. They need a little help. Write down everything except asides (actually, some asides are worth recording as well). Say the words as you write them. This is also a device for slowing yourself down. Most of us tend to talk far too fast—at least about mathematics. Slowing yourself down and writing deliberately will help you to keep your handwriting clear and will make the lesson as a whole appear to be neat and clean. If the appearance is neat and clean then perhaps the ideas are neat and clean—at least that’s what you want the students to think.

The flip side of the last paragraph is that the tendency to talk too rapidly may cause you to write too rapidly (and therefore sloppily). Thus periodically checking the quality of your handwriting on the blackboard can serve as a means of telling whether either your verbal or written delivery is too speedy.

Let me reiterate one of my most fundamental precepts. There is a real psychological barrier for the instructor to overcome when learning blackboard technique, and voice control. When we understand very deeply what we are talking about, then it all seems quite trivial. We can convince ourselves rather easily—at least at a subconscious level—that it is embarrassing to stand in front of a group and enunciate whatever mundane material is the topic of the day. Thus we are inclined to race through it, both verbally and in the way that we render it on the blackboard. Be conscious of this trap and do not fall into it. I have never been criticized for being too clear, whether I was giving a calculus lecture to freshmen or a seminar lecture at the Mathematisches Forschungsinstitut Oberwolfach. Slow down. Be deliberate. Enunciate. Explain.

Many of us, at the beginning of the class, rattle on verbally at some length before we finally persuade ourselves that we had better start writing something on the board. Please don’t do this. Start writing the material from the very outset. If you want the students to notice it, and write it down, and get it straight, then you had better set the example by writing it.

Writing material neatly and slowly is a subtle way of telling the students that this material is important. If you are taking the trouble to write it down deliberately, then it must be worth writing deliberately. Conversely, if you scribble some incoherent gibberish, or scribble nothing at all, then what signal are you sending to the students?

2.4 Body Language

If you skulk into your classroom, stand slouching in front of the class with a furtive and disreputable expression, wear slovenly clothing, and give off a ripe odor that permeates the room well into the fifth row, then you are sending numerous negative signals to your class. It sounds trite to say it, but dress neatly and attractively when you go to teach. Stand erect and look dignified. Master the basics of personal grooming. Attending to these mundane matters really does make a difference. If you want the class to behave as though you are in charge, then look and act the part. It is true that Grigory Rasputin broke all these precepts and still managed to rule a country, but he had many other talents that are foreign to most mathematicians.
2.4. BODY LANGUAGE

Think about the last time you attended a colloquium party or other mathematics social function. In my experience, there is always a group of eight or ten people who don’t have the good sense to go home at the propitious time (obviously the only reason I know this is that sometimes I’ve been one of those people). These stragglers usually pull chairs into a ring and sit and stare at each other for an hour or so before the host finally says a cheery, “Well gang, time to go!” This “circle of death” is a strange and embarrassing artifact of mathematical—and perhaps academic—social life. One cannot help but suppose that people who would display this sort of social ineptitude might be equally gauche in other circumstances, such as when they teach. Do endeavor to be aware of the world around you. Endeavor to get to know your students, and to get to know who they are. Try to understand their needs, and their goals. Use your body (as well as your wit) to show them that you are there to help them learn the material, to be their guide in this new territory.

A friend of mine, fresh from Romania and with minimal English skills, once sat in a lunch room in Chicago endeavoring to formulate his order. He stared forlornly at “Ruben Sandwich”, “Chile Size”, “Three Pigs in a Blanket”, “Montecristo Sandwich”, and other solecisms on the menu trying to determine what in the world they could be. His dictionary was of little help. Finally he found a menu item that he could translate: “bacon, lettuce, and tomato sandwich”. He happily signaled the server and painstakingly ordered “a bacon, lettuce, and tomato sandwich and a Coke.” The server scribbled down the order and drawled “Ya want mayo on your BLT?”

Has it ever occurred to you that we treat our calculus students in just the way that that server treated my Romanian friend? The students are struggling with the chain rule and the mean value theorem and related rates in just the way that my friend struggled with the concept of Ruben sandwich, and we often answer (because we are so accustomed to the ideas) “Ya want mayo on your BLT?” It is important that we develop some sensitivity to the student’s point of view, his values, and his vocabulary. We need not admire these attributes, but we must deal with them. That is the teacher’s job.

Not long ago I asked a young assistant professor how he taught the Mean Value Theorem to freshmen. He began by saying, “First I tell them about the Axiom of Choice …” I could not resist interrupting and asking “Why?” He seemed to have some vague idea that the Axiom of Choice had something to do with the completeness of the real numbers (this turns out to be a misapprehension). But so what if it does? I wouldn’t say a word about the completeness of the reals when explaining the Mean Value Theorem to eighteen year olds, much less about the Axiom of Choice. In fact I’m quite sure that I wouldn’t even prove the Mean Value Theorem. I would concentrate on getting them to understand what the theorem says. In short, while my friend would be asking the freshmen “Ya want mayo on your BLT?” I would be serving them a nice hot sandwich on a platter.

It is best—really—if your students can think of you as a human being, part of the same species to which they belong. Intellectually you may be on an entirely different plane. Nevertheless, when you are not running bulls in Pamplona or swapping gestalts with the Maharishi, you are in fact just the calculus instructor.
It is an important job, and you should endeavor to carry it out with dignity and professionalism.

2.5 Homework

In most lower-division courses, and many upper-division ones, it is by way of the homework that you have the greatest direct interaction with your students. When students waylay you after class or come to your office hour, it is usually to ask you about a homework problem. This is why the exercise sets in a textbook are often the most important part of the book (textbook authors do not seem to have caught on to this observation yet) and why it is critical that homework assignments be sensibly constructed.

Let me stress again that I am not trying to sell you a time-consuming attitude or habit. If you take twenty minutes to compose a homework assignment then you are probably taking too much time. But consider the following precepts:

- Do not make the homework assignment too long.
- Do not make the homework assignment too short.
- Check over the problems you assign to confirm that there are no notational or obvious typographical errors. (Students can waste great amounts of time trying to fathom typos that are trivial to you and me. As a result, they become quite frustrated and angry. Doing this sort of checking shows them that you are on their side.)
- Be sure that the assignment touches on all of the most important topics.
- Be sure that the homework assignment drills the students on the material that you want them to learn and the material that you will be testing them on.
- Make sure that at least some of the homework problems are graded.
- Plan ahead. The exams that you give should be based only on material that the students have seen in the classes and in the homework.

If homework does not count and is not graded, then students will not do it. That is a fact. I realize that many of us have neither the time nor the inclination to spend long hours each evening grading homework. Many universities and colleges these days simply do not have the resources to provide enough graders for lower-division courses. But there are compromises that you can make. For example, you can tell the students that, of ten problems on the homework assignment, just three will be graded. But don't tell them which three. This device will force most of the serious students to do all the homework problems, but it requires much less grader time to get the grading done.

If the last suggestion will not work for you, then you can give weekly quizzes that you yourself will grade. The amount of your time involved will be little,
and it is a device to force students to keep up with the work. Incidentally, this
device also gives you a gentle way to keep your finger on the pulse of the class.

Consider the implementing following policy to help get your students more
interested in doing the homework. Students can and do benefit from collabor-
ated, just as we mathematicians do in our research. While you probably do
not want to encourage collaboration on exams, you may wish to encourage it on
homework. Of course I'm not talking about "I'll copy yours this week and you
can copy mine next week." Instead, I'm talking about an intelligent exchange of
information among equals.

Some studies have shown that one reason that Oriental students in this coun-
try tend to do very well in their mathematics classes (and there are surely many
reasons) is that they work in groups. More precisely, they first work hard as
individuals. Then they get together and compare results. In short, they collab-
orate in much the same way that mature mathematicians collaborate. They are
willing to say, "I can do this but I cannot do that. What can you contribute?" At
the same time, the studies indicate that certain other elements of the student
population are either loath to work in groups or are unaware of the benefits of
this activity. These strata tend to do poorly in mathematics classes. See [TRE]
for details.

Some of the more interesting teaching reform projects, including those from
Harvard and Duke, are specifically designed to encourage students to learn math-
ematics through group activities. Reports on these experiments are encouraging.

If you do decide to encourage group work in your classes, then you will have
to make peace between said collaboration and your grading policies. If homework
is not collected, then there is no problem and you can separate the good students
from the bad through exams and quizzes. If instead homework is collected, then
you will have to consider carefully how to tell whose work is whose, or at least
how to divide up the credit.

2.6 Office Hours

At most universities the instructor is required to hold three or more office hours
per week. Choose three hours that are convenient for you or convenient for
the students or both. Monday/Wednesday/Friday at 11:00 A.M. is, on most
campuses, one of the most popular times for classes. If you schedule your office
hour at that time then many students will not be able to attend. One good
strategy is to stagger your office hours, so that they are at different times on
different days. Another is to make an office hour from half past the hour to half
past the hour, so that a student's class is likely to overlap only half of it rather
than all of it.

Of course you cannot select a time for office hours that will please everyone,
so don't even attempt to do so. Set your office hours, and announce them,
and explain to the students that you can make appointments for those who
cannot attend the regularly scheduled hours. Such an announcement will not
appreciably increase the number of visitations from your students, and it is just
good business to set such a policy.
Promise students that you will be there during your office hour. And be there. Students should be made to understand that they need not wait for a natural or personal disaster in order to come to your office hour. It is perfectly all right for a student to come to your office hour and say "I don't get problem 6," or "The chain rule makes no sense to me."

During your office hour, you will usually not be overwhelmed with students (except perhaps just before an exam). In fact it is a general rule of thumb that, the larger the class, the smaller the percentage of students who will come to your office hour. But those who do show up will appreciate your attentions. Of the hours that you have designated, you can spend some of them catching up on your correspondence, making up the next homework assignment, or reading the Notices or the Monthly or the Mathematical Intelligencer.

If you have sufficient space in your office, it is a good idea to have a table and a couple of chairs set up in a special part of the office—away from your desk and your papers and books and personal artifacts—where you will consult with students. What is the reason for this affectation? First of all, you don't want students inadvertently walking away with your papers or your correspondence. Second, you don't want them spilling coffee on your latest manuscript or your new book that you purchased at great expense from Marcel Dekker. Third, students are by nature careless. They may put their feet on your desk or use your telephone or grab your fountain pen. Rather than appear to be an old fuddy-duddy and constantly be scolding, it is so much easier to have a special venue in which to "hold court".

When a student comes to my office expressing befuddlement over a particular type of problem, I have a powerful and decisive weapon that I unleash. I begin by asking, "Do you have a half hour or so?" If the answer is "Yes", then I sit the student down and say, "Try a problem of the kind you are having trouble with. When you get stuck, tell me." Of course the student invariably gets stuck, and I give him a little help. I might need to intervene three or four times during the first problem that the student does for me. But the second problem may require only two interventions, and the third only one. By the time the fourth problem rolls around, the student's newfound confidence is irrepressible, and the transaction is a great success. The student goes away pleased and happy that he has now mastered a heretofore mystifying mathematical idea. Of course I always tell the student, "If you get home, and you find that you are still confused, then come back and we'll do this again."

On days when your office hour is not crowded, and you only have a couple of customers, I highly recommend that you try this teaching technique. It's good business, and it always produces satisfied customers. Word gets around in the class that the professor is not such a bad guy after all. Perhaps, as a result, a few other students drop by for help.

You want to convey to students that the office hour is a particular time that you have set aside for them. If you consult with students while sitting at your desk and glancing at your mail, or scribbling notes for an upcoming seminar, or answering phone calls, then you in fact will not convince them of your dedication. Instead, if you hold court in the special part of your office that you have set aside for consultations, then your students will understand that this time is theirs. If
you really want to do it right, then let your voice mail pick up on your phone calls during your office hour. For those sixty minutes, give yourself to your students.

The office hour is your opportunity to get to know at least some of your students personally. Of course I do not mean by this that you should get involved in their personal lives. Problems about their love lives or their parents or their social diseases should be referred to the professional counselors that are on the staff of every college. What I mean is that you should take the opportunity to get to know some of your students as people, and to let them get to know you as well.

This activity has several beneficial side effects, both for you and for them. When you are lecturing, you can have certain individuals in the room in mind as you formulate your remarks. You can make reference (without mentioning any names) to questions that came up during office hour. It is reassuring to the average student (the type that does not go to office hour) to know that conscientious students (the type that do go to office hour) have some of the same questions that they have.

This point is in fact worth developing. Some components of teaching may be compared with certain components of psychotherapy. One big aspect of therapy—certainly an aspect that is exploited by popular psychology and self-help books—is to assure the patient that he is not alone. There are thousands of people with exactly the same problems, suffering in just the same ways. And they have been treated successfully.

Just so, when you teach you must give both tacit and explicit reassurances to students that their questions and confusions are not theirs alone. An eighteen year old is scared to death that he is the only person in the room who doesn't understand why the numerator in the quotient rule has the form that it has—or why it does not seem to be symmetric in its arguments. Such a student would not dare ask about it in front of a room full of his peers. The student may not even be sure how to articulate the question, so surely will not want to flounder about in front of the entire class. At the same time the student may be afraid to come to your office hour and, alone but in your august presence, ask for a clarification.

Thus you must signal to students that questions are a good thing. When a student asks a question in class that might be of general interest, I not only repeat it but I often state that I am glad this question was raised. I carefully record the question on the blackboard. Several people have visited me privately, I add, and asked variants of the same question. If there is a question that should be asked but has not been, then I ask it myself. I say that if this point is unclear to them (the students) then they should come see me in my office hour and get it straightened out. You don't need to give away door prizes to drum up business at your office hour. However, it is psychologically important for students to know that you are available, whether they actually come to see you or not.

I have said repeatedly in this book that persistence is an important attribute for the successful teacher. Another such attribute is patience. If a student finds the nerve to ask a question in class or during office hour—even if it is a question that I have answered before, in fact even if I have answered it several times before—then I treat the questioner with respect and I pay due homage to the
question and I answer it. I never say, "I've already answered that question. Go home and read your notes." Such a rejoinder would be counterproductive, and would discourage further question-asking in class.

I often announce to my classes that students may drop by my office even when it is not my office hour. If I am not busy, I'll be happy to talk to them. In practice, this charity does not appreciably increase the flow of business. There are always students who strictly respect your designated office hour and there are always those who drop by whenever they please. But making an announcement of this nature is one of those little details that contribute to a favorable student attitude. For it sends a signal to the class that you care, and that you truly want to help them learn mathematics. If you do make such an announcement, be courteous to those who take you up on it. If you are busy and must send the student away, do so with respect and suggest another time for the student to return.

I once had a colleague who, whenever a student would show up at his door, would crawl under his desk until the student went away. This is certainly a memorable way to deal with students, but not one that I would recommend. When a student comes to your office, make him feel welcome. Act as though you are happy that he dropped by. Endeavor to adopt the same cheery tone that you would assume if a good friend paid a surprise visit. Such an action on your part will put the student at ease, and will make the transaction go smoothly and productively.

The office hour is a way to step out of your role as instructor and let the students know that you are a person. It is a way to become acquainted with some of your students. Any good public speaker "works the audience" before his speech. Holding your office hour is one way to work the audience. You will also get a feeling during the office hour for how the class is doing, what problems and concerns have arisen, how the pace is working. It is wrong, and self-defeating, to view your office hour as a dreary duty. It is a teaching tool that you should use wisely.

2.7 Designing a Course

Many of us never have the privilege of actually designing a course. Instead, we are assigned to teach prepackaged courses that the department has already assembled. This will especially be true if your teaching load is primarily "service courses": precalculus, calculus, linear algebra, ordinary differential equations. In non-service courses—upper-division courses or courses that are taught for majors—you may in fact have considerable discretion as to what you will include in the course, and how you will organize it. In what follows, I shall draw a sketch of what input you may have into the structure of a course, and also what input you may not have.

If you are teaching one of the prepackaged service courses, then certainly the content will be pre-specified. And, like it or not, you had better stick to the syllabus or outline that the department provides for you. Your students are taking this course only because it is required for their major. If you are an
3.4 BREAKING THE ICE

It doesn't require much effort to show that you care about the class. If they do well on a test, celebrate with them. If they do poorly, commiserate with them. If they cannot do the homework, then help them with it. If they can't understand the concept of orientability of a surface, explain it to them. Show them a Möbius strip. They have probably all seen Möbius strips before, but never realized that they had a serious use. The point is that you should share their pain. You are there to help them learn. Act like it.

I once witnessed a conversation between two mathematicians—-one American and one German. The first man was discussing John James Audubon, the American naturalist. The second was discussing the Autobahn, the famous high-speed German highway. Each scholar prattled away for ten minutes or more, discussing his own topic, blissfully unaware that his interlocutor was discussing something entirely different. How was this possible, you may ask? Well, how many mathematicians do you know?

Has it ever occurred to you that, sometimes, when you are talking to your calculus students, you are no different than the savants in the last paragraph? Here is the poor student trying to tell you that he can't even understand the logic in problem #6 on the homework, and you are busy trying to explain to him the circumstances under which the mean value theorem fails. Such a situation doesn't make you either a bad person or a bad teacher. But it certainly makes you less than optimal in helping your student to learn. Part of caring is to step out of your own shoes and, as much as possible, step into the student's. It may be a new experience for you, but it is one that you should seek.

3.4 Breaking the Ice

The first day of class is simultaneously a day of happy anticipation and a day of stress. It is the first of these (assuming that you like to teach) because you are, after a restful summer, jumping into something that you enjoy and that you do well. It is stressful because you don't know what this new group of students is going to be like, or whether they will play ball with you, or whether you can get through to them.

I am a teacher of long experience. On days of exceptional hubris, I convince myself that I am rather a good teacher. Yet most semesters, especially in the fall, I meet a new class with new students and I have to demonstrate to these people that I'm a good guy. We begin as total strangers, and my goal is to turn us into a working group. Usually this takes a while—often several weeks.

Since I so enjoy a class once we have all become friends, I find the period of tooling up to that happy steady state generally too long and too painful. What usually happens is that there is a period of two to five weeks during which the students look at me as though I am from Mars. They don't laugh at my jokes, they don't answer my questions, they don't seem to take me very seriously. If the class is to be a success, then some magical thing must happen to change everyone's attitude.

You should consider ways to make yourself seem like a human being to your students. Being playful, or impish, or making fun of yourself, is certainly one
technique for accomplishing that goal. If that doesn't work for you, or makes you feel uncomfortable, then try something else. Read them some history. Tell them of Bishop Berkeley and his doubts about calculus. Tell them about fractals, or dynamical systems, or wavelets, or why\(^1\) mathematicians don't get the Nobel Prize.\(^2\)

Find some way to open up to your students so that they will open up to you. Some instructors hide their unease behind regimen. They take roll, or put together a seating chart, or ask each student to introduce himself. This routine is fine if you are comfortable with it. My view is that you should show students from day one that you are a person, and that you are going to spend the term doing your best to communicate with them. I don't think that taking roll is a good way to send that signal. It is better if you tell them what the course is about, or describe your grading policies, or give them some clues as to what you are like.

To repeat an important theme of this book: If your students are not talking to you it is probably because you are not talking to them. Set the tone on the first day. And never forget it.

3.5 Why Do We Need Mathematics Teachers?

When I wrote the first edition of this book, I had trouble formulating a cogent answer to this question. We all believe that teachers are necessary. Society must believe it too, for it deigns to pay (not very generously) a great many teachers. But why are teachers necessary? Two colleagues (Gary Jensen and Meyer Jerison)—both wiser than myself—have supplied a striking and memorable answer:

The teacher

1. Sets a pace for the students;
2. Teaches students to read (mathematics);
3. Helps the students to become engaged in the learning process.

There is considerable wisdom in these simple observations. Let us consider them one by one.

1. Watch a young person—or even an old person—attempt to learn to play the piano without benefit of a teacher. Such a person usually has neither the

\(^1\)There are actually several versions of this story. The so-called French/American version is that a mathematician (Mittag-Leffler) ran off with Nobel's wife. The Swedish version is that Alfred Nobel was a practical man of the world who wasn't aware of mathematics as a discipline.

\(^2\)If you are tired of the standard Nobel Prize story, then tell the lesser known story of the Mittag-Leffler Prize. Mittag-Leffler set it up, of course, to spite Nobel. He mandated that the medal would be twice as large, and the award twice as grand in several notable aspects. It was only awarded twice because Mittag-Leffler invested the funds in the Italian railroad system and German World War I bonds.