1.3 PREPARE

you feel that you are above the task of teaching this course, then your students will get the message immediately. And what are you accomplishing by evincing these attitudes? Does it make you feel superior? More accomplished? More secure? More important? It should not. Teaching is an essential part of what we do. Society sees us as teachers and, increasingly, the university sees us as teachers. Why not see yourself as a teacher? It will increase your self-esteem. And it will increase the esteem that others have for you. Being a good teacher does not detract from your mathematical attributes. It augments them.

1.3 Prepare

Some people rationalize not teaching well by saying (either to themselves or to others), “My time is too valuable. I am not going to spend it preparing my calculus lecture. I am so smart that I can just walk into the classroom and wing it. And the students will benefit from watching a mathematician think on his feet.” (As a student, I actually had professors who announced this nonsense to the class on a regular basis. And, as you can imagine, these were professors who royally botched up their lectures on a regular basis.)

It is true that most of us can walk into the room most of the time and mostly wing it. But most of us will not be very successful if we do so. Thirty minutes can be sufficient time for an experienced instructor to prepare a calculus lecture. A novice instructor, especially one teaching an unfamiliar subject for the first time, may need considerably more preparation time. Make sure that you have the definitions and theorems straight. Read through the examples to make sure that there are no unpleasant surprises. It is a good idea to have a single page of notes containing the key points. To write out every word that you will say, write out a separate page of anticipated questions, have auxiliary pages of extra examples, have inspirational quotes drawn from the works of Thomas Carlyle, make up a new notational system, make up your own exotic examples, and so forth, is primarily an exercise in self-abuse. Over-preparation can actually stultify a lecture or a class. But you’ve got to know your stuff.

I cannot emphasize too strongly the fact that preparation is of utmost importance if you are going to deliver a stimulating class. However it is also true that the more you prepare the more you lose your spontaneity. You must strike a balance between (i) knowing the material cold and (ii) being able to “talk things through” with your audience.

My own experience is that there is a “right amount” of preparation that is suitable for each type of course. I want to be confident that I’m not going to screw up in the middle of lecture. But I also want to be actually thinking the ideas through as I present them. I want to feel that my lecture or class has an edge. It is possible to over-prepare. To continue to prepare after you have already prepared sufficiently is a bit like hitting yourself in the head with a hammer because it feels so good when you stop.

You must be sufficiently confident that you can field questions on the fly, can modify your lecture (again on the fly) to suit circumstances, can tolerate a diversion to address a point that has been raised. The ability to do these things
well is largely a product of experience. But you can cultivate this ability too. You cannot learn to play the piano by accident. And you will not learn to teach well by accident. You must be aware—in detail—of what it is that you are trying to master and then consciously hone that skill.

If you do not prepare—I mean really do not prepare—and louse up two or three classes in a row, then you will experience one or more of the following consequences: (i) Students will take up your time after class and during your office hour (in order to complain and ask questions), (ii) Students will stop coming to class, (iii) Students will complain to the undergraduate director and to the chairman, (iv) Students will (if you are really bad) complain to the dean and write letters to the student newspaper, (v) Students will write bad teaching evaluations for your course.

Now student teaching evaluations are not gospel (see Section 2.9). They contain some remarks that are of value and some that are not. Getting bad teaching evaluations does not necessarily mean that you did a bad job. And I know that the dean will only slap me on the wrist if he gets a complaint about my teaching (however, if there are ten complaints, then I had better look out). Finally, I know that the chairman will give me the benefit of the doubt and allow me every opportunity to put any difficult situation in perspective. But if I spend thirty minutes preparing each of my classes then I will avoid all this grief and, in general, find the teaching experience pleasurable rather than painful. What could be simpler?

As well as preparing for a class, you would be wise to debrief yourself after class. Ask yourself how it went. Were you sufficiently well prepared? Did you handle questions well? Did you present that difficult concept as clearly as you had hoped? Was there room for improvement? Be as tough on yourself as you would be after any exercise that you genuinely care about—from playing the piano to engaging in a tennis match. It will result in real improvement in your teaching.

Read your teaching evaluations (Section 2.9). Many are insipid. Others are puerile. Most, however, are thoughtful and well-meant. If ten of your students say that your writing is unclear, or that you talk too quickly, or that you are impatient with questions, then maybe there is a problem that you should address. Teaching is a yoga. Your mantra is “Am I getting through to them?”

It is a good idea to try to anticipate questions that students will ask. But you cannot do this artificially, as a platonic exercise late at night over a cup of coffee. It comes with experience. Assuming that you have adopted the attitude that you actually care whether your students learn something, then after several years of teaching you will know by instinct what points are confusing and why. This instinct enables you to prepare a cogent lecture—to know what to emphasize, where to slow down, where to provide extra examples. It helps you to be receptive to student questions. It helps you to have a good attitude in the classroom.

An easy way to cut down on your preparation time for a class is to present examples straight out of the book. The weak students will appreciate this repetition. Most students will not, and you will probably be criticized for this policy. On the other hand, it is rather tricky to make up good examples of maximum-minimum problems or graphing problems or applications of Stokes's theorem.
1.4 CLARITY

It can be time-consuming as well. If you need more examples for your calculus class, then pick up another calculus book and borrow some. Develop a file of examples that you can dip into each time you teach calculus. You will learn quickly that making up your own examples is hard work. Do you ever wonder why most calculus books are so disappointing? All right, you try to make up eight good examples to illustrate the divergence theorem.

1.4 Clarity

When you teach a mathematics class, clarity (or lack thereof) manifests itself in many forms. If you are the most brilliant, and even the most well prepared, mathematics teacher in the world, but you stand facing the blackboard and mumbling to yourself, then you are not being clear. If instead you shout at the top of your lungs so that all can hear, but your handwriting is cryptic, then you are not being clear. If your voice is clear, your handwriting clear, but your blackboard technique nonexistent, then you are not being clear. If your voice is beautiful, your handwriting artistic, your blackboard technique flawless, but you are completely disorganized, then you are not being clear. If you speak clearly, write clearly, have good blackboard technique, are well organized, but speak with a foreign accent, then don’t worry. You are being clear.

Here is the point: Mathematics is hard. Do not make it harder by putting artificial barriers between yourself and your students. If you are shy and simply cannot face your audience, then perhaps you chose the wrong profession. More seriously, be extremely well prepared. Make yourself confident. Calculus is one of the most powerful analytic tools that has ever been created. It is a privilege to be able to pass it along to the next generation. Be proud of what you are doing. It is no less an event for you to teach the fundamental theorem of calculus to a group of freshmen in the 1990s than it was for Archimedes to teach his students how to calculate the area inside a circle.

I have atrocious handwriting. When my departmental librarian got her first written message from me she thought it had been written in traditional Chinese characters. But when I lecture I slow down. I write deliberately and clearly. I want my audience to understand me and to respect me and I take steps to see that this actually happens.

Suppose that you are in the middle of a lecture and you are making a very important point. How can you drive it home? How can you get the students’ attention? We all know that students drift into a malaise in which they are copying and not thinking (after all, we were once students and did the same). How do you wake them up? It’s easy. Pause. State the point clearly and simply. Write it clearly and simply. Say “This is important.” Repeat the point. One of Mozart’s most decisive tools in his compositions was repetition of a particularly beautiful passage. We can benefit from his example.

Ask whether there are any questions. Repeat the point again. Assure students that this point will be on the exam, and that it will come up over and over

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2Surely you have heard the old saw about the concepts passing from the professor’s lecture to the students’ notes without ever entering the students’ brains.
2.15 Small Classes vs. Large Classes

We all know, deep in our guts, that small classes are a much more stimulating venue for learning than large classes. After all, in a small class students can participate, they can feel much more a part of the process, they can get to know the instructor personally, they are more comfortable asking questions, and they experience a lot of one-on-one interaction.

All true, but (as noted in the last section) there are no studies that show significant improvement in learning, performance, or retention when students are taught in small classes rather than large classes. In fact the main difference that can be objectively verified is that students in small classes feel better about the class, feel more empowered, and have higher self-esteem than those in large classes.

This is certainly a situation in which we instructors must train ourselves to separate objective fact from self-evaluation and intuition. When we argue (with the dean, for example) for smaller calculus classes, we are motivated in part by a concern for student welfare, in part by a distaste for teaching large lectures, and in part by a desire to find objective reasons for hiring more faculty.

If the conclusions of the objective studies are valid, then perhaps we can learn something from them that will inform the way that we teach large lectures. I’m sure that there are few preachers who want smaller flocks, who would prefer to preach to an audience of 30 rather than 300. I wonder why that is so? Can we learn ways to make our students in a calculus class of a few hundred feel more involved, have a better opinion of themselves, feel good about the learning process?

The final word on these questions is certainly not in. But simply being aware of the information in this section and the last should give you food for thought the next time you face down a large class. What is it that makes teaching a large class seem to be difficult? Is it the size of the room, the population of students, the use of a microphone, the feeling that there is less room for error, or what? What can we as math teachers—not preachers or performers—do to make a large class still seem like a family? What can we do to make a large class still feel like a place where learning is taking place?

2.16 Problem Sessions, Review Sessions, and Help Sessions

At many big universities, the large thrice weekly lectures in a lower-division math course are supplemented by once- or twice- weekly “problem sessions” or “help sessions”. Usually the lectures are delivered by a professor or instructor while the help sessions are staffed by graduate student teaching assistants (TAs).

Imagine that you are the graduate student in charge of a problem session. It is easy to fall into the trap of not taking the work very seriously. After all, student attendance at these sessions is poor in general and spotty at best. Students seem to be inattentive and their questions are often puerile. But the quality of
any class or help session is largely influenced by the attitudes and efforts of the person in front of the room. If your attitude is to treat the help session casually or carelessly then you will get correspondingly disappointing results from the students. Consider giving weekly quizzes, sending students to the board, and other devices for livening up your problem session (see also Section 3.12). I wish to concentrate here on more mundane matters.

It is arguably more difficult to conduct a good problem session than to give a good lecture or class lesson. For the problem session presents all the difficulties of a class period, and more. At least in a class you are in complete control of the order of topics and can, if you wish, present them from prepared notes. In a problem session, if you really let the students ask what they wish, then you must be ready for anything. And you must be able to think quickly, on your feet, of the best way to present any given topic, give a hint on any problem, or handle any point of confusion. In a class or lecture you can always pull rank and say, “There is no time for questions now. See me in my office hour.” (I don’t recommend that you say this very often, but it is an option that is available). But help sessions are for questions.

If you are a novice, then it is probably safest to view the help session in the most naive way. Your role is to help students do their homework assignment for that week. Thus your preparation for a help session might consist of working all the homework problems for the week, or at least staring at them long enough to be sure that you know how to do them.

Be certain that the techniques that you present are consistent with those used in class and in the book. Some professors require their TAs to attend their classes, just to insure this consistency. Such a professor might even do a spot check of the grader’s work, or drop in on help sessions to see how things are going.

I know of at least one professor who works closely with his grader and his TAs by attending, once per week, each problem session for his class accompanied by the grader! This requires some extra effort on everybody’s part, but it shows real consideration for the student who has questions about the way that his homework was graded (or how the class, as a whole, is being conducted). It goes without saying that in order to use this device to good effect the professor will have to be well-coordinated with the grader on how he wants each homework assignment graded.

When you are helping with a homework problem that is to be handed in, don’t give away the store. One reasonable answer to the dreary question, “How do you do number 14?” might be, “I’ll do number 16 for you, which is similar.” Another reasonable answer might be, “I’ll get you started. You do the rest.” A third is, “Here is an outline of the basic steps.” The truly skillful instructor will turn this question-answer session into a team effort. Gently goading the students with his own prompts and questions, this instructor will resist simply doing the requested problem for the students. The trouble with just solving the problem—and nothing more—is that only the requestor and perhaps a few others will be paying attention. If instead the instructor can generate some repartee, and can get the students to want to pitch in, then there will be considerable student interest and a number of class members will learn from the experience.
There are subtle psychological forces at play in the scenario just described. If each student is worried about protecting his turf, and simply does not want to share what he knows, then you will have a hard time generating useful dialogue in your problem session. If instead the atmosphere is one of learning being a sharing activity, and of giving knowledge in expectation of receiving knowledge, then the problem session can be a worthwhile and nurturing experience for everyone. (We all know of mathematicians who collaborate easily and well, and of others who seem to be thoroughly incapable of collaboration. Perhaps these differences reflect attitudes similar to those being described here.) Of course you as the TA or instructor must set the example. If the signal you send is that you are not willing to help, that you are not willing to share, that instead you are like the oar master on an ancient galley, then you will get little in the way of cooperation and sharing from your students. If instead the example you set is one of patience and giving and caring, then you are likely to be the beneficiary of an enthusiastic response.

The advice to the TA (five paragraphs ago) to work all the homework problems the night before a problem session is one that I tender hesitantly. I never do this, but I've been teaching math for twenty-five years. I am rarely surprised by any question in a calculus class or help session and, even if I am, I can usually slug my way through whatever new features are present. If I am at a review session for an exam and a student presents a really difficult question then I always have the option of saying, "That's an interesting question, but one that could never be put on the test. Let's discuss it privately."

In your first few years of teaching you will have to strike a balance between being thoroughly prepared (by working all problems in advance) and spending too much time on preparation (see also Section 1.3). Just remember that a large part of your job is (i) to show the students how to do the problems and (ii) to persuade the students that the problems are doable (by ordinary mortals). If you fumble around and act baffled by the problems, then you are presenting a poor role model and, more to the point, doing your job badly. Students find appealing the fact that I can do all the problems and that, moreover, I invariably know where the difficult spots are and can help them to chart their way through them. This ability can only come with experience. It is the model that you should strive to attain.

2.17 Transparencies

I have already touched upon the topic of overhead transparencies (Section 2.3). With the use of transparencies, you can cover more material than you could by just using a blackboard. By using several overhead projectors, you can create an ambience similar to that achieved by several blackboards. By using color, overlays, photos from books, data printouts, computer-generated graphics, histograms, and the like, you can put on a dazzling display of information.

Many of the principles governing good blackboard technique also apply to overhead slides. But, in this somewhat different environment, they take on a new form. Of course you must be organized and write neatly. Be sure to write
experience is someone worth listening to. Find a mentor, ask him questions, and listen to the answers. And then use this input to craft your own style of teaching.

3.12 How to Ask, How to Answer

If a pollster asked the average American voter, “What do you think of the upcoming election?” then the resulting answer would probably not be very enlightening. If you turn to your calculus class one day and say, “OK, now we’ve covered Chapters 3 and 4—any questions?” then you will get a bunch of blank looks. By the same token, if a textbook salesman hands a new calculus book to a math professor and says, “What do you think?”, the professor will probably say, “I dunno; they all look the same to me.” By the same token, students come to professors with questions such as, “Like, you know; I don’t think I understand any of this stuff we’re doing.”

It is a strange facet of the human condition that most of us don’t know consciously what we think about most things most of the time. A skilled questioner learns to ask specific questions in order to obtain meaningful answers. Rather than asking your class if there are questions about Chapters 3 and 4, ask them instead if they are comfortable with the chain rule, or if they can do related rates problems, or falling body problems. The material in a person’s memory is hung on hooks. You must reach for those hooks to get useful answers to your questions.

The same principle applies when you are holding a review session—for a midterm exam, let’s say. If you are serious, if you really want to help the students, then it is simply not good enough for you to stand before the students and say, “Any questions?” They do not know what they want to ask. And, even if they think they know, they are timid about doing so. You must prompt them: “Do you understand integration by parts? Can you do partial fractions? What about the u-substitution? Is Section 7.5 confusing? Was the second homework assignment particularly difficult?” You, the instructor, must understand that your having said these things will (i) jog their memories, and (ii) make it OK for them to ask about these topics. I find that it breaks the ice for me to write a list of topics on the board. This is just one way to get the “review session ball” rolling. Remember: You must poke the students and prod them and, if necessary, embarrass them a little. Never forget the psychological aspects of teaching.

We implement these dicta naturally when writing an exam. You would never set an exam question for freshmen that said, “Tell everything you know about differential calculus.” Instead you ask very specific questions. You want to train yourself to do the same when talking or lecturing to students. More, you want to train yourself to do the same in reverse when you are trying to elicit questions from students.

There is a gentle art of getting your students to pose questions. And I don’t mean questions like, “Will this be on the test?” I mean the kind of meaty, well-thought-out questions that we all live for. Perhaps the most common com-
plaint that I hear from disillusioned mathematics instructors is that they cannot
develop any participation from their lower-division classes (see Section 4.5 on
Frustration). The matter of garnering good questions is a non-trivial issue, and
one to which an entire separate book could be devoted. You are going to have to
find methods that suit your personality, and your teaching style, and that work
for you. (See the Appendix to this section for some specific suggestions on how
to increase student participation and inject some life into your class.)

The devices that you use can be quite simple. For example, giving a good
quiz once or twice per week is a device for focusing student attention on some
particular issues. The quiz is a little bit like a traffic officer pulling you over and
threatening you with a citation. We are all aware—in a general sort of way—of
the traffic safety laws. But if a cop gets in your face and starts telling you things
that you are doing wrong then suddenly the penny drops.

The devices that you use can also be complex. You could have each student
develop an ongoing, long-term project. Such a project might have the property
that it must include material based on the ideas from each week of the course.
And each student must be prepared to report to you at any time on the status
of his project.

You may very well think that quizzes are too trite and semester-long projects
are too massive for you to consider. Fine. I use quizzes frequently in my own
course, and I'm frankly too lazy to do semester-long projects. Finding a way
to get students to participate is something that you must do for yourself.
Consider wheedling, threatening, cajoling, joking, challenging, priming. You can get
through to your students by making them like you, or by scaring the hell out of
them, or by conning them, or by being gruff with them. I am not necessarily
recommending any of these. But if you want to be an effective teacher then you
must find something that works for you.

As you experiment with ways to liven up your class, bear in mind the nature of
the enemy. One enemy is that young adults, for the most part, are quite unsure of
themselves. Unlike an experienced mathematician, who in effect makes a career
out of asking (often stupid) questions, the student is deathly afraid of looking
silly in front of his peers. He is not intellectually mature, and not experienced.
He is not expert in the art of discourse (see also Section 3.14).

This last point is worth developing. If you have survived in the academic
game, then you have learned to ask questions. You would never go up to a
member of the National Academy of Sciences and say "Duh. I was trying to prove
an interior regularity theorem for the Laplacian, and I just cannot seem to do it.
I tried integrating by parts, but I couldn't decide what to do with the boundary
term." Your friend the National Academy member would—justifiably—probably
conclude that you were an idiot. A safer way to pose the question would be: "I've
been thinking about interior regularity for the Laplacian. I know the classical
ideas, but what is the modern approach? What would be a general context in
which to fit this type of question?"

If you know something about elliptic partial differential equations, then you
are probably not sent into paroxysms of ecstasy by the second question either.
But it certainly sounds more intelligent than the first. And it gives the questioner
some room to maneuver. Students simply don't have this skill at discourse, so
they resort to the obvious subterfuge—they clam up. Part of your job as teacher is to help your students learn to engage in scholarly discourse. Help them to ask questions. If a student asks a weak question, help him to turn it into a better one. Try to create an atmosphere in which you and the students are co-explorers. Convey that you will sometimes make false starts, and so can they. It's a knack, but you can learn it.

Another enemy, besides the observed fact that students are uncertain and don't want to talk, is that mathematics can be (it is not by nature) a dry, forbidding subject. Part of your job as teacher is to make the subject come alive and to motivate the students to want to learn the material. This book supplies a variety of techniques for achieving that goal (Sections 1.7, 1.12, 3.1, 3.3, 3.5, 3.7, 3.12, 3.14).

APPENDIX: SOME SUGGESTIONS FOR ENCOURAGING CLASS PARTICIPATION

This appendix contains several techniques, drawn from the literature or from my own experience, for bringing your class to life. Take them for what they are worth. Some may appeal to you, and some may not. But reading about them may give you ideas of your own. Note that the activities discussed here are designed for classes of manageable size. They do not lend themselves well to a large lecture of 350; see Section 2.14 for a consideration of techniques suitable for that environment.

In lower-division political science courses, it is common for the instructor to begin a class by saying, "Today we are going to be a medieval village. Who wants to be the mayor? Who wants to be the executioner?" And so forth. It is quite natural for a mathematician to react to that type of classroom activity with derision, to observe that it appears to be childish and non-productive. Perhaps, but such devices are a wonderful way to get students involved with the subject matter. What can we do in our math classes that will (i) teach the students something of value and (ii) get them involved with the subject matter? Here are some possible answers.

1) Get students to go to the blackboard. I have noted in Section 3.1 that this is not necessarily the most efficient use of time. But it is a way to get the students to participate. If you wish, and if it is feasible in your learning environment, you could record problems on the board before students come into the classroom. Those who wish can go to the board—even before class begins—and work problems. To avoid having the same old students monopolize this activity, you could institute a rule that no student may work a problem at the board twice in one week. Of course the entire class should discuss the various solutions that are so recorded.

2) Have students prepare oral reports or mini-lectures. This activity is usually best reserved for the last part of the semester, when everyone is tired and
students are receptive to a change of pace. Since most of the students will be inexperienced in activities of this nature, I recommend that you assign students to each give a fifteen minute lecture on a very specific topic. Time considerations show that this activity is only feasible in a rather small class.

3) Have students take turns writing and grading quizzes. It might be appropriate to assign a team of three students to each quiz. Not only will this activity cause the students to think critically about the material that they are studying, but it will also imbue them with an appreciation for the sorts of things that you, the instructor, must do.

4) If a student cannot do a problem, and brings this fact up in class, then have him go to the blackboard and explain what he tried and where he got stuck. It is certainly true that some students will be too shy to pull this off, but most students will be secretly thrilled to be treated like fellow scholars. You can orchestrate a similar activity for a student who does know how to do a problem.

5) Use “Minute Notes”. These work in the following manner. Once every week or so, ask students to jot down on a slip of paper anything that is bothering them—problems that they cannot do or concepts that they cannot understand or anything else that pertains to the class. You give them just one minute for this task (hence the name). Do it at the beginning of the class hour, and collect the notes right away. Read them on the spot. You will suddenly have a much clearer picture of what is going on in the class, what concerns the students have, where you should go from there.

Perhaps more importantly, you will have given the students a feeling of empowerment. You will have helped them to understand that their input is a constructive part of the class. After a few weeks of Minute Notes, you will generally find that students are much more willing to raise their hands in class and make meaningful contributions to the learning experience.

6) If you are truly daring, then you can design your course so that it is more like a literature course. That is, you give the students regular reading assignments and homework assignments, but you do not lecture directly on a linearly ordered sequence of topics. Instead you come to class each time with an air of, “Well, what shall we talk about today? Who would like to begin?” The idea is that your classroom is a marketplace of ideas. You need to really know your stuff, and have an engaging manner, to pull this off. But it is bound to be great fun.

7) Have guest instructors. To use this tool well, you must work closely with the guests to be sure that they will talk about material that is salient to the class, and will present it at an appropriate level. If you think of the fourteen weeks (give or take) of your course in the same way that I have discussed single lectures or classes (see Section 3.7), then having guest instructors is a way to prevent your course from being an “uninflected monotone”. You can also consider roles that graduate students, teaching assistants, and “teacher's aids” (i.e.,
3.12. HOW TO ASK, HOW TO ANSWER

teachers in training doing their practicum in your class) might play in livening up the atmosphere.

8) If you have the resources, and the breadth of acquaintance, or if your department has the contacts, you could bring in guest speakers from industry or government or business. Imagine a calculus class in which you bring in someone working on the NASA space station project to talk about how calculus is used to design the work platform for the engineers in space (I'm not making this up; there really is such a project). Students would really wake up and smell the coffee when confronted with such a class experience.

9) This technique was devised by Jean Pedersen. She asserts that it works extraordinarily well for her. It is called the method of "mathematical POST-IT® notes".

We all know that POST-IT® notes are those little squares of colored paper that easily can be affixed or un-affixed to a document for the purpose of making remarks or memos. The idea for the application of these devices in a math class is that the professor comes to class with a tablet or two of these notes, each having the professor's name (or some other identifiable epithet) stamped on it. Whenever a student asks a good question (not "Will this be on the test?" or "What is this stuff good for?" or some pseudo-question that the student just cooked up), then he is rewarded with a POST-IT® note. "So what?" you ask.

When the next exam comes around, the students are instructed to bring their POST-IT® notes along. They are to affix them to the front of the exam that they hand in. The student then receives two extra points (or some number to be pre-determined) for each POST-IT® note.

Reports are that, when this policy is announced in class, it is as though a jolt of electricity has run through the room. Suddenly hands are waving in the air, and previously uninterested students become the life of the party.

Now let me be the first to admit that this teaching device, like any other, is not perfect. Some students who are already alienated will become more alienated if they are unable to garner any POST-IT® notes. Other students may object that they are being treated like children. Think carefully before you try this, or any, new technique.

10) I have saved the most frivolous suggestion for last. Although you probably will not choose to use it yourself, it may suggest analogous techniques that more naturally suit you and your classroom. And, although the technique is a bit silly, it is currently in use by at least one successful math teacher.

On the first day of class the instructor announces that he is very embarrassed to report that he simply cannot spell. Students should feel free to correct his dreadful spelling. Then he begins to lecture, spelling "line" as "lien" and "book" as "buk". Students are so delighted confidently to be able to correct the professor's spelling that participating constructively in the mathematics portion of the course becomes very natural.
I find this last technique of deliberate misspelling to be a bit dishonest, but it's hard to argue with success. In my own classes, I endeavor to create the feeling that we are all creating the lesson together. I do this with a constant line of patter, much like that used by a magician or an illusionist. With this technique, I have the students talking all the time as well. If a mistake is made in class, then it is our mistake, and we fix it together. If a problem is solved correctly, then that is our shared triumph.

The key to bringing your class to life is to become involved with the students and to make learning a shared activity. Perhaps this is one of the great lessons of the reform movement. It is not an ideal learning environment to have the teacher as stick-man preaching before an audience of sponges. Learning should be done symbiotically, and it is up to the instructor to structure his class accordingly.

3.13 Teaching with the Internet

The Internet is a marvelous tool for making information available to a large body of people quickly. For example, if a mathematics department subscribes to an electronic journal, then as many people in the department who wish to do so can read the journal at the same time. Those who want to study a paper carefully can download it, compile it, and print it out. When you prove a new theorem, you can post your abstract (or your entire paper) on an electronic bulletin board. Your results are then instantly known around the world.

The Internet is also a useful teaching device. Create a Web page for your class. Put the class syllabus on the Web page. You could have a page about prerequisites for the course, or ancillary reading, or ways to prepare for exams. Post homework assignments and due dates on your class Web page. Put information about upcoming exams there. If you need to write up a correction to something from class, or disseminate a list of errata to the text, or post homework solutions or exam solutions, then the Internet is just the ticket.

I once read a proposal for an "Internet Mathematics Curriculum". The premise was that, at certain universities with a great many part-time and commuter students, absenteeism is a problem. Students have families and jobs and cannot always make it to class. In the electronic age, modes of communication are changing—so why not take advantage? The proposal was that the professor would still give his lecture, and those who could attend would do so. But there would be assigned note-takers who would post official notes on the Internet. The Internet could also be used to cut through the problem that students will not—or are too shy to—participate in class. The math class would have its own electronic bulletin board(s), and students could post their queries there—anonymously or not. Other students, or the professor or the TA, could answer the queries as they saw fit. Since many students have the same questions, this use of a bulletin board would allow the professor to use his time more efficiently.

The proposal that I just described was not funded. In fact it didn’t even make the first cut. I think there is real merit to some of the ideas just described. But
I also think that the concept of an Internet University abrogates much of what
the learning process is all about. Classes are held for a good reason, and it is
this: Many things that we do in life have a ceremonial aspect. We hold funerals
to come to grips with someone's passing, and to create a sense of closure; we
have graduation ceremonies to pause to think about an important moment in
a young person's life; we select people for prizes (the Nobel Prize, the Cannes
Film Festival Award, etc.) in part to recognize talented individuals and in part
to ponder the human condition and what we are trying to achieve. Just so, we
hold classes so that the students will take an hour, go to a special place, sit
in a controlled environment, and think in a focused manner about a particular
subject under the guidance of an expert. If this were not as important as you
and I know it to be, we would not do it.

My point is that Internet classes, while they may have their place, eliminate
what is powerful about attending a class. Glancing at prepared lecture notes
for your calculus class on your computer screen is (for the student) a bit too
casual, and too much like turning on the radio. The student attempting to
learn in this manner could be interrupted by the telephone, the doorbell, a pot
boiling over, a baby crying, or any number of other exigencies (again, this is
why traditional classes are a good thing). A mature and disciplined person with
suitable scholarly training might be able to learn successfully from an Internet
class. I'm not so sure about inexperienced eighteen-year-old students.

You can use the Internet as the nerve center of your class, to keep everyone
informed of up-to-the-minute information and last-minute changes, to post new
homework assignments, to post grades, to change your office hours, to give last
minute room or seating assignments for the upcoming exam, and so forth. The
concept of fielding questions over the Internet, or with e-mail, is a fascinating
one. The one obvious impediment is that most students don't know how to enter
mathematics using the keyboard.\footnote{The software product NetTutor by Link-Systems is designed to cut through this problem.
It presents the student with a white board on which to write his query by hand. Or else
the student can click on icons to pull down mathematical symbols. The student can submit
a question anonymously or not. The professor can answer questions in real time or at his
convenience—and he can do so publicly or privately. The professor also can, with little effort,
create a database of frequently asked questions that he can allow the students to access.}
This certainly is more efficient than trying to remember to photocopy the information and bring it to class, it avoids the class
time wasted when you distribute handouts, and it is more permanent (that is,
the material can always be found right there on the Web for the duration of the
term).

I believe that the full picture of the value of the Internet as a teaching device
is yet to be determined. But I caution you against thinking that it can be a
substitute for classroom learning.

3.14 The Art of Discourse

Ask yourself this question: If a student has a successful and fulfilling college ed-
ucation, then what does he take away with him? Twenty years after graduation,
what does that student still retain? What intellectual framework does he have to build on?

Comic Don Novello, in his role as Father Guido Sarducci (on the television show Saturday Night Live), gave the following answer. "If you majored in Economics, all you remember is ‘Supply and Demand’. If you majored in French, all you remember is ‘Pariez vous Français?’ If you majored in Physics, all you remember is ‘Every action has an equal and opposite reaction.’" (He might have added, “If you majored in math, all you remember is ‘Take the exponent and put it in front.’”) So Father Sarducci proposed that people not spend four years and $100,000 on a traditional university education. If this is all you are going to retain, argued the good cleric, Father Sarducci will teach it to you in five minutes—and charge you much less. He called his solution “Father Guido Sarducci’s Five Minute University.”

We who devote our lives to university teaching hope fervently that there is considerably more to higher education than Father Sarducci’s droll diatribe would suggest. In this section I am going to endeavor to say what that “more” is.

The naïve answer to the question “What does a student get from his education?” is that the student receives career training. Certainly career training has significant value, and should not be dismissed lightly. But if we take the long view then we can see a larger picture. We can see depth and texture. What a student ought to take away from college is (1) critical thinking skills and (2) knowledge of and experience with discourse (see also the discussion in Section 3.10). These two aspects of education are essential, and they are not disjoint.

In college, a student declares a major. And that is the area in which the student obtains advanced training. But most of the student’s courses are not in the major. In those other courses, the student is learning philosophical discourse, humanistic discourse, the discourse of social thought, and scientific discourse. The student is learning different modes of critical thought.

For example, Renaissance philosophers considered the questions, “What is the world we see and what is the world we experience and what is the world that is actually out there? Are they one and the same world? If not, then how do they differ? And how can we tell?” Renaissance mathematicians studied algebraic equations. Renaissance musicians studied the lute. All of these are valuable avenues of inquiry, and they are all quite different. An important part of gaining an education is learning about these different modes of thought.

When we teach undergraduates—especially lower-division students—we are primarily teaching non-majors. It can certainly inform our teaching, and remind us of what we are about, to be cognizant of our goals when we teach. When you teach calculus to a pre-medical student, or finite math to a business student, you are endeavoring to acquaint him with modes of mathematical thinking, with our special method of reasoning and analysis.

In fact, when I teach my undergraduates, I have in mind a much larger and more ambitious goal. I want to teach my students that the world need not be a place in which they are passive observers. They need not spend their lives “letting things happen.” Put in other words, we do not—at least should not—live in a world in which some nebulous other people generate ideas, and hold
office, and make decisions. In fact it is we who are to become educated, to
assume the positions of leadership, and to make the important decisions. To my
mind, this is the role in society of an educated person. Perhaps we instructors
have, in our own lives, realized this truth. But we should determine to pass it
on to our students.

Surely it is more constructive, and more fulfilling for everyone involved, to
bear these thoughts in mind as we lecture these unformed lumps of clay. Do not
view the teaching process as a sorry labor—akin to shoveling out the Aegean
stables. You are not trying to turn these eighteen and nineteen year olds into
little mathematicians. Instead, you are trying to educate them, to stretch their
minds, to teach them to analyze and to think critically.

3.15 What about Research?

Mathematicians of my generation were taught, both explicitly and subliminally,
that there is only one thing worth doing—and that is mathematical research.
Teaching and curriculum and departmental service and university service are all
done well, but not among serious mathematicians. I have no reason to believe
that more recent generations have been trained any differently.

OK. I believe it, you believe it, we all believe it. But the observed fact
remains that there are precious few of us who can sustain a rigorous career
of mathematical research from cradle to grave. On a festive occasion when
he turned 50, A. Besicovitch (1891–1970) commented that, “two thirds of my
mathematical life is over.” Twenty years later, someone observed to the contrary
that Besicovitch had written more than half of his papers—many of them among
his best—since turning 50. He hastened to remind Besicovitch of the foolish
remark that the great man had made at age 50. Besicovitch’s reply was, “Well,
numerator was correct.” A great and inspiring story, but few mathematicians
can claim that their best work was done between the ages of 50 and 70. We get
tired, we run out of steam, we want for ideas.

As you read the words of this book on how to teach, you may be thinking,
“This is all just dandy. But I am a research mathematician; that is what I do.
I have neither the time nor the inclination to worry about teaching. And I’m
frankly surprised that Krantz has the time to do so.” I don’t know whether I
truly have the time. My research program is going rather well right now, thank
you very much. But I made the time to write this book. I think that teaching is
important.

I also think that our profession would be healthier if there were room for
us to pursue other activities besides research. To put it in different words: If
you can do research, if you want to do research, if you’ve got the ideas and the
drive and the stamina, then that is what you should do before all else. But,
instead of spending the waning years of your mathematical life staring forlornly
at the wall, why not think about the curriculum, or think about creating a new
course, or think about writing a textbook, or think about how to improve your
teaching, or how to help others in the department improve their teaching? We
all know that we get some of our best mathematical ideas while mowing the lawn