of exceptions that are connected with dealing with people. I have used the “incomplete” here as but one example of the problems and potential enigmas that can arise. Your department probably has set policies, or at least guidelines, for handling incompletes. Become acquainted with the routine procedures before you give your first "I".

4.5 Frustration

One of the most commonly heard complaints of college mathematics instructors, especially experienced instructors, is this: “Math 297 is a prerequisite for the course that I am teaching yet the students don’t seem to know anything from Math 297.” A variant of this is “My calculus students cannot add fractions” or “My calculus students don’t know how to expand \((a + b)^2\).”

Indeed, these are valid complaints. It is also valid to complain about the high cost of living, or about death and taxes. The peccadillos described in the first paragraph are facts of life and we as math instructors must deal with them. The truth is that we instructors think about math all day long, every day. We see the entire curriculum as a piece. For the experienced math instructor, there are no seams and creases between linear algebra, calculus, differential equations, and so forth. We swim effortlessly through the ideas, using whatever tools are needed. (By the way, if this doesn’t exactly describe you then don’t panic—I’m using a bit of poetic license here.) Students are different. They think about math when they are in the math classroom and (one hopes) for a few designated hours outside the classroom, but they are not inured in the subject.

So what is the point? It is simple. If you are in the second week of freshman calculus and you need to add two algebraic expressions that are fractions, then gently remind the students how to do it. If you need to expand the expression \((a + b)^2\), then say, “You remember how this works—right?” After a few gentle reminders, most of the students fall into the flow and they will remember how it goes.

Take a break and watch the “Tonight Show” or “Late Night”: Listen to the monologue. If the host is going to crack a joke about someone slightly less famous than Bill Clinton, then he gently reminds the audience who it is that he is talking about. It’s just good sense. These television hosts can be even less sure of how well informed their audiences are than we can be in our math classes. They guarantee that their viewers will understand by providing a bridge.

Contrast these recommendations with the following rather common alternative. The professor needs to add two fractional algebraic expressions, so he just barrels through it (rapidly and without comment, as one would do for a colleague). After a few moments some hands are raised, some hesitant questions are asked, and it soon becomes clear that many students are lost. The professor says, “What is the matter with you people? This is high school stuff. Am I a

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1The Bank of America in Westwood Village, Los Angeles used to regularly place an advertisement in the UCLA student paper each spring. The purpose of the ad was to encourage members of the latest graduating class to consider a career with B of A. The ad read in part “applicants must be able to add fractions.” So it is not just math teachers who are plagued by this problem.
baby sitter or what?" (I'm not making this up; I have colleagues who do just this.)

In my view the behavior of the mathematics instructor in the last paragraph is the mathematical analogue of shooting one's self in the foot. The professor (perhaps unconsciously) sets up a situation for failure. And there is no good or useful point to it. It not only sets a bad tone for that day, but also for the remainder of the course. The instructor needs only to expend just a little extra effort to anticipate these pitfalls, and to devote a few seconds to allaying them. And it does a world of good.

In fact, for me, helping the students recall how to add fractions or to expand quadratic expressions (or any other analogous elementary operation) is a form of protective coloration. It is too easy for me to make errors when doing these elementary operations. If I whip through them, leaving most of the students in the dust, then I in fact increase the likelihood that I will make an error, and also abrogate any sympathy I might have garnered when my error was detected. If instead I slow down and walk the students through the calculation, then it becomes "our calculation". They help me to check it, and the chance of an error is reduced to virtually nil.

As indicated at the beginning of this section, these frustrations also present themselves at more advanced levels—even with math majors. As an instance, linear algebra is often a prerequisite for multi-variable calculus. And well it should be, for matrix language is a natural vehicle for expressing the derivative, the chain rule, and so forth. But it is an artifact of the American mathematics curriculum that linear algebra is often taught in a vacuum. The students have no hooks to hang the ideas on, and they do not remember them very well. There is no alternative, if you want to keep your multi-variable calculus course on an even keel, to giving a whirlwind review of the salient linear algebra ideas as you use them. Here, by "whirlwind", I mean a five or ten minute snapshot, on the fly, of the relevant idea right before it is used.

Here is another way to look at the matter of frustration. You and I have become accustomed, when we visit our physician or our attorney or our psychotherapist, to a certain amount of professional decorum. Often the doctor or lawyer or counselor meets us in a well-appointed office, dressed in a suit or other formal attire, and he exudes courtesy, detachment, and professionalism. Sadly, academics do not seem to have bought into this game. If you had a fight this morning with your spouse, or got a speeding ticket when driving to work, or got a rejection of your latest paper in the mail, then you are liable to take it out on your students. Your office may look like a pigsty and your haberdashery like something from Zola's La Terre. But you have set a standard for your students and if they don't meet it then you may lose your objectivity or your patience and you may react.

If you have been dutifully teaching your calculus students for several weeks running, and if their latest midterm shows that they've absorbed very little of your wisdom, then you are liable to vent your spleen at them. You would never expect your family doctor to start hollering at you about losing weight, nor your lawyer to scream at you about paying your taxes on time, nor your psychiatrist to excoriate your for being too neurotic. But you and I sometimes find ourselves
altogether losing it and—more is the pity—giving hell to our students.

Of course there are notable ways in which doctors and lawyers differ from academics. What makes us special is that we endeavor to impart knowledge to our students and we expect them to radiate it back at us. When they fail to do so, then we are disappointed, sometimes angry, and certainly frustrated. What I am suggesting here is that it can serve to your advantage to set yourself apart from your students. Maintain some objectivity. Try not to become emotionally involved. As award-winning teacher David S. Moore observes (see [MOO] as well as Section 3.1), teaching is a job: You prepare your class and you do it. If there are problems, you deal with them. If the students aren't learning then you teach harder. It is part of the academic milieu, and part of our training, to think of ourselves like operatic divas: If things don't go as they should then perhaps a tantrum is in order. Not so. Be strong.

If the students are not working hard enough, nor absorbing the ideas at a pace and depth to suit your ideals, then too bad. But it's too bad for them; it's really no big thing for you. Teaching freshman is like mowing your lawn. No matter how good a job you do this Saturday, you are going to have to do it again next Saturday. Yelling at the lawn doesn't help.

Of course I am disappointed when my students—despite my best efforts—can't do three-dimensional graphing, or can't understand Stokes's theorem, or can't apply the ε-δ definition of continuity. But my job is to teach and I just get in there and do it. If I have to cover a tricky topic twice, or even three times, then that's the breaks. Part of being a successful teacher is gaining your students' trust. Go watch the movie Stand and Deliver about the legendary calculus teacher Jaime Escalante. He was tough on his students. He told them when he was disappointed and he worked them hard. But he never belittled them, and he never lashed out at them. He showed genuine pride and enthusiasm when they did well. The most important thing he did for his students is that he made them believe in him. They worked hard for him because they trusted him.

The frustration problem described here is one of the few in this book that plague the experienced instructor somewhat more than the novice. Novices are usually drunk with youthful enthusiasm for teaching. Middle aged folks like myself are often just tired. We tend to lose our patience, and to forget the struggles of the uninitiated. An instructor who has been dealing with, and teaching, the ideas for twenty years cannot understand why students don't remember what they have already seen once. Once! The key to success here is to try to develop (or remember) a little sensitivity to the point of view of the students.

As a closing thought, consider the following. If your students are not speaking to you then it is probably because you are not speaking to them. You may be lecturing at them, you may be exhorting them, you may be talking down to them, or you may be venting your spleen and verbally abusing them. But you are not relating to them as people. You are not teaching. Try it—you'll like it.
4.6 Annoying Questions

At several junctures in this book I have mentioned some spine-tingling, bone-chilling, conversation-stopping questions that students can and will ask. One of these is, “What is all this stuff good for?” Another is, “Will this be on the test?” Another is, “Why don’t you prepare your lectures more carefully? You are wasting our time.”

If you are asked the third question then the fault probably lies with you. You should have done a better job preparing your class. If things are really going dreadfully, you might say to the class, “I apologize. This class is going very badly. Let’s quit for today.” Nobody will take this amiss, and it is probably the most diplomatic way out of an uncomfortable situation—but do not use this device more often than about once per semester. The best policy is to use forethought to prevent such an encounter.

Dealing with the first two questions, and others like them, is something that you will learn to do through bitter experience. In America in the 1990s, we endeavor to educate a broad cross section of the population. We cannot assume, as perhaps a don at Oxford could one hundred years ago, that our students are at the university primarily to learn to become refined citizens—and that they are happy to consider whatever we set before them. In particular, today’s students are prone to challenge what we are doing. It is a part of your job to be prepared to answer their challenges. The challenges are not generally hostile, but having respect for your audience requires that you be prepared to provide a thoughtful response. If you accept my premise—repeated throughout this book, but particularly in Section 3.14—that getting an education is learning the art of discourse, then you should set an example for your students. If one of them poses an intelligent, well-thought-out question (even though it may be one that you don’t particularly want to hear), then you should endeavor to answer it in a manner that is both correct and intellectually stimulating. Now let me say a few words about the particular queries pinpointed at the outset of this section.

Let us consider the question, “Will this be on the test?” One option that you have for an answer is the obvious one. Tell them that it will be on the test and then, indeed, put it on the test.

Now let us look at the opposite situation. If you are going to present something to your students and have no intention of testing them on it, then you have two choices. You can tell them up front that they will not be tested on it—they should just sit back and listen. Or you can tell them that they will be tested on it and then don’t test them on it. Know consciously which choice you have made before you proceed.

If you choose the first route indicated in the last paragraph, then you might put the exercise in context like this: Explain that some ideas are difficult and deep. It requires several exposures to such an idea before it begins to make sense. This is an opportunity for the student to begin to ponder something important. Students are pleased to be treated like fellow scholars, and will usually act accordingly.

If you choose to tell the students that $X$ will be on the test but in fact you have no intention of putting $X$ on the test ... That is OK, but don’t over-use
this privilege. It will irritate the students and could damage your credibility.

The question "What is all this stuff good for?" is treated in Sections 1.7, 1.11. You, the mathematician, can get so wrapped up in your mathematics that such a question can catch you entirely off guard. Spend a few moments arming yourself against it.

The main point is this. You are not lecturing fellow mathematicians, who are inured to your point of view. You are lecturing students. Students will challenge you and ask questions. Some of these questions are difficult. If you want to retain the students' respect, then you must be prepared to deal with their queries and to understand their point of view.

4.7 Discipline

One hopes that, in a college environment, discipline will not be a big problem. But there are difficulties that can arise (see Section 3.11 for an extreme example).

In a big class, with two hundred or more people, student talking can get out of hand. Many students read the newspaper, or knit, or eat their lunches, or write letters to their friends. Some students, when the lights are turned low, engage in romantic activities. Students come in late and leave early. Students sleep.

I see no point in making a spectacle over a student who is not causing a disturbance. If a student is quietly eating lunch, then that is no problem for me. If a student comes in late or needs to leave early and does so in an orderly and non-disruptive fashion, then I let that student alone. (A truly courteous student will tell me in advance that he needs to leave early or come in late, and I am suitably appreciative. If you find this custom attractive, then tell your class that you want to be notified in advance of such temporal irregularities.) To make a scene will only alienate the whole class.

How should you handle a student who causes a disruption? First try something gentle like, "OK, let's quiet down." Another technique is to simply stop talking until you have everyone's attention. If one or two applications of "nice" is ineffectual, then come down on the offender quickly and sternly. Example: "Mr. Trump, if you want to talk then please leave the room." or "Ms. Lewinsky, everyone else is here to learn. Please keep quiet." If you deliver these injunctions firmly and with confidence, then they will have a chilling effect.

Often you can, arrange for the other (non-offending) students to be the bad guys. If you simply sit down and wait for silence and cooperation, then the other students will "shush" the offenders. You can just provide a modicum of stern looks.

William James was both a father of twentieth-century psychology and also a renowned teacher. When he felt that his class was not cooperating—either inattentive, or talkative, or simply dull—he would fold his arms and lie down on the floor. After a while, the class would fall into puzzled silence. Then he would announce, "Education goes both ways. You have to participate too!" And he would try again. As you can imagine, a clever teacher could make something like this device into a productive game with his class. I have a special place on the
wall of my classroom where I ceremonially bang my head when my students are being slow. You should create your own tricks—ones that suit your personality and your style.

On those absolutely rare occasions when a class is beyond control, you might throw down your chalk and say something like, "This class is hopeless. I'm through for today. We'll try again on Friday." I have never used this last device, and I fervently hope that I shall never have to resort to it. But somehow it gives me strength to know that it is at my disposal. If you do take this extreme measure, you had better let your chairman know what you have done.

I have seen large mathematics classes (of about 400 or more students) which looked like a cross between a rock concert and a Hieronymous Bosch painting (see also Section 2.14). Private conversations and mini-dramas were taking place all over the room while groups of students roamed the aisles. The professor stood at the front of the room, bellowing away on his microphone, while a small percentage of the students attempted to learn something. Such a situation is plainly unacceptable. Nobody can ask a question, nor can there be any interchange of ideas, in such an atmosphere. Certainly the question of learning the art of discourse is all but absurd in this context. Generally speaking, a situation like this comes about gradually over the course of the semester. It happens because someone (most likely the professor) lets it happen.

Some professors prefer to deal directly, and in advance, with the discipline problems connected with large lectures. On the first day of class in a large lecture, these instructors tell the class that large classes present special organizational problems. In order to make the experience as beneficial as possible for everyone, the instructor goes on to prescribe certain rules of behavior in the classroom. These include no eating, no talking, no reading of the newspaper, no coming in late, no leaving early, and so forth. Remember that the first few lectures of your class are your chance to set the tone. You may wish to take the opportunity (gently) to "lay down the law".

Of course if you are going to use the stern system described in the last paragraph—and it is a perfectly reasonable one—then you must follow through on it. If a student breaks one of the rules you have laid down, then you must call him on it: "Mr. Goering, I said no eating in class." or "Ms. Flowers, if you need to leave class early then you shouldn't come at all."

Because you are an authority in your field, because you give out the grades, and because you hold sway over students' lives, you have both moral and de facto authority in the classroom. As a result, if you comport yourself like a concerned, dedicated professional, then you should have relatively few disciplinary problems. If you nip behavioral problems in the bud, and handle them with dispatch, then they will not get out of hand and your classes will go smoothly. But your antennae should be out for trouble. You will figure out quickly who the wise guys and troublemakers are. When they start to rattle, you start to roll.

Some instructors, in extreme problem situations where there are continuous interruptions and talking during a large lecture class, enlist a confederate. The confederate can sit in the back of class (unidentified to the students) and take notes on student behavior when the (official) instructor's back is turned. That way, after consultation with the confederate, the instructor can be absolutely sure
who is doing the talking, throwing the spit wads, and causing the trouble. And he can act accordingly. I was always impressed by my grade school teachers who had eyes in the backs of their heads. They knew when I was going to misbehave even before I started thinking about it. Most of us have not developed such a skill, and may find it (on rare occasions) useful to invoke the device described here.

Not long ago I had a student come to me and tell me that he'd skipped the previous two weeks of class. But he was now returning and would do his best to catch up. I said "fine"—if he needed some guidance, then he should let me know. Indeed, he showed up in class that day. I began the lecture by saying, "OK, let's have another look at Stokes's theorem." My prodigal friend, who had just been in to see me, said, "Could you give a quick review of this concept?" (I had been discussing Stokes's theorem for most of his two week absence.) I am ashamed to say that I lost it. I said, "No. Not for someone who hasn't been to class for two weeks." The other students supported me in this. I could tell by looking in their eyes. But I felt like a rat. When I conducted my personal debriefing after class, I wondered whether I had done the right thing. It all came out well, because a few minutes later he came to my office and apologized to me, I apologized to him, and everything was hunky dory. In retrospect, I think I should have said, "We've been studying Stokes's theorem for two weeks, and it doesn't lend itself to a quick summary. See me after class if you want more help."

Always remember that you have the power to command respect, but you cannot demand it. If you present the image of an organized, knowledgeable scholar who is trying to do a good job of teaching, then most students will play ball with you. If instead you are a bumbling, unprepared clod who clearly doesn't care a damn for the class, then you can expect like treatment from the students.

I used to have a colleague who handled late arrivals to a large lecture in the following fashion. He would cease to lecture and make a show of timing how long it took the student to get seated. Then he would say, "There are 150 students in this class. It took you 90 seconds to get seated. Thus you wasted 3.75 hours of their time. Now each student here has paid $N$ dollars to take this course. Let us next calculate how many of their dollars you have wasted." And so forth.

Let us consider the effect of this practice. Certainly any student who planned to come in late in the future would wear a bag over his head. But I cannot help but think that this sort of arrogant behavior on the part of the instructor suggests a serious attitude problem. Students will lose respect for an instructor who behaves in this fashion.

As the instructor in a classroom, you are in charge. You have every right to demand a certain type of behavior from students, and to enforce discipline. But you must not, in the course of disciplining a student, diminish his self-respect. Students are young adults, and should be treated as such.

We have all seen parents who cannot control their children. We have also seen 95 pound fathers who hold tremendous sway over their 250 pound linebacker sons. Parents of the latter sort understand the difference between demanding respect and commanding respect. The first is easy, is a convenient way to vent your spleen, and often doesn't work. The second is more gentle. It is an art that you need to cultivate. The techniques suggested in this book should help you in
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that task.

Remember this: You should be conscious of maintaining discipline in class from day one. This is not to say that you should be an unbending authoritarian—far from it. But if you let a class slide out of control for six weeks and then try to use the techniques suggested here (or other techniques) to take back the reins of power, then you will have an extremely difficult and unpleasant time.

The famous mathematics teacher R. L. Moore (see Section 1.12) is said to have once brought a Colt 45 to an unruly math class, set it conspicuously on the table, and then proceeded into his lesson in a room so quiet that one could have heard the sun rise. This technique may have been suitable in Texas fifty years ago. These days, however, I would recommend the use of more civilized techniques for keeping order.

4.8 Mistakes in Class

The most important rule to follow before giving a class is to prepare (Section 1.3). How much you prepare will depend on you—on your experience, your confidence, your training, and so forth. Being fully prepared gives you the flexibility to deal creatively with the unexpected.

But nobody is perfect. No matter how well prepared you are, or how careful, you will occasionally slip up. In the middle of a calculation, a plus sign can become a minus sign. An $x$ may become a $y$. You will say one thing, think a second, write a third, and mean a fourth. It is best if you can handle these slips with a flair, and particularly without sending the class into a tailspin.

I endeavor in my classes to create an atmosphere in which students are comfortable to shout out, “Hey, Krantz, you forgot a minus sign.” Or, “Is that a capital $F$ or a lower case $f$?” This is a form of participation, and it can be a very constructive one. If you handle these situations badly, then students will be less inclined to ask questions or to approach you on other, more important, matters.

If mistakes are small, and occur in isolation, then they will not damage the learning process. But if they are frequent or, worse, if they snowball, then you will lose almost everyone, give a strong impression of carelessness, set a bad example, and (to oversimplify) turn off the class.

You may endeavor to bail out of an example that you are lousing up by saying, “Well, this isn’t working out. Let’s start another example.” It won’t work. This is in the vein of two “wrongs” not making a “right”. The only solution here is not to make mistakes and to handle those that you make anyway with a certain amount of finesse.

However: If you can see that the example you are working on is getting out of control, if you know that it is going from bad to worse, that you are so bolloxed up that you will be unable to bail out of it, then what do you do? Do not spend the rest of the hour trying to slug it out. Doing so is uncomfortable, counterproductive, and will not teach anyone anything. Instead apologize, say that you will write up the solution and hand it out next time (or put it on the class Web page!), and move on. My advice here may seem to fly in the face
of Section 2.8, and to contradict the last paragraph, but it is only meant for extreme situations. Making mistakes is one of the surest ways to lose control of a class. It is the mathematical analogue of an equestrian letting go of the reins. Strive not to do it.

Besides preparing well, there are technical devices for minimizing the number of errors that you make. When I am working an example in a lower-division class, I pause frequently to say, “Let’s make sure this is right” or, “Let’s double check this step.” I often pick out a student (who I know will respond well) and ask him whether that last step was done correctly. This procedure provides a good paradigm for the students. It also allows note takers to catch up and allows the bright students to strut their stuff in a harmless manner.

One of the most common ways that students make mistakes in their work is by trying to do too much in their heads. Therefore you should set a good example. Write out all calculations. Point out explicitly that you have had many years of experience with this material yet you still use lots of parentheses and write out every step.

4.9 Advice and Consent

If your students take a shine to you, and many of them will, then they will view you as looming larger than just “the math teacher”. They will come to you for advice on all sorts of things, from the purchase of a computer or computer algebra software, to advice on the purchase of a car, to advice on how to handle their parents, or advice on very private matters.

A good rule of thumb for you as teacher is to stick to things that you know. You are probably well qualified to give guidance about math books, which section of calculus to sign up for next semester, which computer to buy for which purposes, or whether MACSYMA is preferable to Mathematica. If you are an auto buff you could give advice about wheels. But being in a position of authority and being asked for advice by a semi-worshipful student is heady stuff, and you had better be careful.

When you are advising students as to which math class to take, it is easy to fall into the trap of unintentionally (or, more is the pity, intentionally) criticizing your colleagues. The practice of this indiscretion is unfortunately rather common. Please do not fall into it.

The students are your clients, but in some sense you work for their parents (since they probably pay the freight). You are almost certainly out of place to advise your students on how to behave toward their mother and father. Do so at your own risk.

When a student starts asking you about private matters then you are in dangerous territory. It is often difficult to discern the difference between (i) a student asking how to deal with a significant other and (ii) a student making a come-on. Unfortunately, sexual harassment and political correctness, are a part of life these days. Defending yourself against an allegation of either is one of the loneliest and most miserable battles that you may ever have to fight. It can threaten your self-respect, your career, and your marriage. A word to