incorrectly, it is possible that he will want to see you give the correct solution. Otherwise interest is minimal. Best is to make the written-out exam solutions available to all class members (in a posted copy, or in a hard copy that you distribute in class, or on the Web). Always insist that, before a student comes to you to complain about how a problem is graded, the student must first read your solution. You cannot imagine how much time and travail this practice will save.

You also need to get on with the course you are teaching. Make it clear to the students that you welcome their comments about the exam. Make yourself available during office hours, and even have extra office hours for discussing the exams if appropriate. But don’t waste valuable class time hashing over an exam that has been taken and graded. Such an exam is basically a dead issue, and it is time to move on.

Generally speaking, it is best to try to deal with specific student questions about the way a specific person’s exam was graded in a private, one-on-one fashion. You should never handle such complaints in front of a class. It is also a bad idea to handle them in front of a group of six after class. This is a personal matter. Treat it like an appointment with a physician. See also Section 2.11 on grading.

2.11 Grading

The pot of gold at the end of the rainbow, from the student’s point of view, is the grade at the end of the course. Grading is a multi-parameter problem. The students want to be treated fairly, yet they want to feel that the course has substance. They want to be enlightened, yet they want (to some degree) to be delighted, to be entertained. They want to respect you (the instructor), but they want to be your friend. There are a variety of devices for making your grading scheme palatable (without being essentially more lenient) to students. What is the most evenhanded and efficient way to determine grades?

I have used a number of grading schemes successfully, and some unsuccessfully. I would like to record a few of the former here—merely for the reader’s delectation. My main goal in formulating my grading policies is to make the greatest number of students feel that they have been treated fairly (and, not incidentally, to reduce student complaints). This does not mean that I am a lenient grader, nor that I give away grades for no special reason.

*Always* tell students on the first day of class, and in your syllabus (see Section 2.12), how you will grade the course. You want this to be a matter of public record. If students complain about your grading practices, and there will occasionally be some who do, then you have your public statements to fall back on. And don’t lie. If you say that you will grade according to a certain scheme—with exams worth so much and homework worth so much and so forth—then do so. If you say in your syllabus that you will grade on a curve, then do so. If you say in the syllabus that you have an absolute grading method (90% is an “A”, 80% is a “B”, etc.), then stick to that.

You may wish to consider in advance how you will handle students who are
distracted about their midterm grades. One method that usually works for me in borderline cases is to say to the complaining student, "If these few points really make a difference in your course grade at the end of the semester then let’s discuss it at that time.” This arrangement usually makes everyone happy, and very few students will take you up on the offer.

If a student comes to complain about a grade, then show the student some courtesy. If you cannot come up with a cogent reason for the way that you graded an exam or a problem, then that is your fault. Rethink your grading practices. Never fall back on your august position as your first line of defense. You show the student absolutely no respect by saying, “that’s the way I graded your test and I’m the boss.” That is not how you would wish to be treated. You can always turn a session of “Why was this test graded this way?” into a favorable transaction. What does it cost you to give the student a few extra points if the points are merited?

However never penalize a student for being honest. If the student comes to you and says, “You added up my points incorrectly. I should have received an 87 instead of a 90.” then just send the student home with a little praise for being so perceptive. Tell the student that if you ever inadvertently give him too few points then he should feel free to approach you at that time as well.

It is tempting, especially when you are a new instructor, to endeavor to take an “organic” approach toward grading. Students are very receptive to the instructor who says, “I try to grade on a subjective system. If your strong grades are on the midterm exams, then I downplay the homework and the final. If your work shows improvement, then I take that into account when I determine your grade. I try to emphasize everyone’s strengths. I am your friend.” This approach works well in the short run. It is a good opiate—for you as well as for the students.

One weakness of the organic grading method is that it is intrinsically unfair. A student who does his homework with friends (and who therefore, despite his own personal lack of understanding, gets good homework grades), but who takes his exams alone (and gets poor exam grades because of his lack of true understanding) could still garner an “A” or “B” in the course.

Perhaps a more pragmatic and immediate liability of the “organic” grading method is that, if a student complains about his course grade, then you have nothing to fall back on. You cannot show him your calculations and you cannot show where his score fits on a histogram. The trouble with an intuitive methodology is that you cannot explain it or defend it. Even though it sounds a trifle cold, you are much better off with an objective system of grading. In the end, everyone is more comfortable with a dispassionate approach. And, in the rare event that you have to defend yourself to the chairman, or to an angry parent, or to the dean, or even to a colleague, you will be prepared.

One device that I have used in large calculus classes (see also Section 2.14 for additional thoughts on grading in large classes) is the following: I tell students that I determine their grade by weighting their midterms as 50% of the grade, the final as 30% of the grade, and the homework as 20% of the grade (for example). But the caveat that I throw in is that anyone who gets an “A” on the final gets an “A” in the course. This assumes, of course, that the final exam is
2.11. GRADING

comprehensive. Thus if a student comes to me during the term and is distraught about his homework grade or his midterm grade, then I can simply enjoin that student to do well on the final. In fact not many students pull their grade up with the final exam (never more than 5%) and this simple device helps to keep morale high.

As with many items in this book, I offer the last tentatively. I have had some of the brighter students complain about this policy: "Why should some jerk who didn’t work all semester be able to pull off an ‘A’ at the last minute just by cramming?" I patiently explain that it is virtually impossible for such a "jerk" to pull off such a miracle, that the purpose of the policy is to help and to offer encouragement to students who have been struggling, or for whom this is the first difficult math class. It also sends the important message to students that what is important is that, by the end of the semester, they ultimately master the material. Some faculty have told me that the skewed value system that the just-described policy implies is sending a counterproductive message to the students. My primary motive in formulating the policy was to give the students hope, and to quell their misgivings and their fears; and not least I wanted to minimize their complaints. In a substantial course—say real analysis—in which it takes time for the students to internalize the ideas, this policy helps students to show (integrated over time) what they have ultimately learned.

Sometimes you must change a student’s grade—either on an exam or in a course. Perhaps you made a clerical error in recording the grade, or you made an error in grading a problem, or you were the victim of any number of other human frailties. Do not be afraid to change a grade when it is merited. However: You do not want to develop the reputation among students as an instructor with whom grades can be negotiated. I’ve had this rep, and I don’t know how I got it. But there was a time when, the day after an exam, 85% of my students lined up outside my office to take turns sluggin it out with me—point by point and problem by problem—over their exam grade. I felt at times as though I should buy stock in the Kleenex Company. This process is unpleasant and (can be) degrading both for you and for the student. Doing a careful job of grading in the first place, and posting carefully written solutions for students to see, can help to assuage much of student discomfort with grades.

Make the student read your (posted) solution before you agree to talk about the grading of a problem. Many times I, as an inexperienced instructor, have spent fifteen minutes haggling with a student over a problem only to realize that the student had not read the correct solution. Once he read it, his objections faded away.

No matter how fair and ethical and “right” your grading methods may be, the way you grade may run afoul of departmental policies. I know many a mathematician who has painstakingly prepared a grading curve for his calculus course and submitted his grades to the department, only to have the Chairman or the Director of Undergraduate Studies call him on the carpet for not following the “approved departmental grading curve”. The usual party line is, “In this department, we recommend that you give 20% ‘A’s, 30% ‘B’s, 40% ‘C’s, and 10% ‘D’s. We discourage ‘F’s.” Of course these specific figures are manufactured, but the scenario occurs all the time—probably more often at large state institutions
(subject to various public pressures) than at small private ones. This is no laughing matter. Many math instructors will just go along with the chairman’s wishes, simply because it is easier than bucking the tide. Others will stand on their right to autonomy in their own classroom.

This is a tough ethical call, and I cannot tell you what is right. You have to live in your department, with the colleagues and the policies that it has. Probably the best advice I can give is that you should find out what the departmental policies are before you begin to teach. If you think that you are going to have trouble living with the grading policies, then discuss them with departmental honchos and try to work out a position that everyone can live with.

It seems to be an increasingly common occurrence (see [WIE]) for a student to come to the instructor after the course is completely finished and say, “I got an ‘F’ in your course. Could we talk about how to raise my grade?” This is like buying a car, signing the papers, making the down payment, driving the car home, and then coming back a week later to see whether you can renegotiate the sale. It makes no sense.

A grade is supposed to be an evaluation of the work that the student performed during the term. When the grade is given, the work (or lack of it) is a done deal. The very notion that this is a point for haggling is a genuine travesty—it shows a true misunderstanding of the university’s mission.

At a prominent university in St. Louis—not my own—there has developed a new process that is called “grieving”. A student who receives a disappointing grade in a class will say “I’m going to grieve this grade.” There is a dean who is in charge of such matters, the student makes an appointment with that dean, and then the young scholar puts on a dog and pony show to convince said dean that the grade is unfair. Then the dean changes the grade! Without consulting the instructor of the course!! Sadly, the practice of grieving amounts to an institutionalization of the wretched behavior that I described in the last two paragraphs. The institution in question is funded strictly according to its enrollment, and it takes great strides to see that its customers are happy. While I sympathize with the school’s plight, I certainly do not endorse its practices.

2.12 The Syllabus (and the Course Diary)

Every mathematics course should have a syllabus. The teacher should give the course a little thought and planning before classes begin. What is the text? What will be covered? What are the prerequisites? How many exams will there be? How will the grade be determined? What is the instructor’s name, office number, phone number, and office hour?

The syllabus should be in outline form—not paragraph form—and display essential information so that it is easy to find. A sample syllabus follows.
Just remember that you are not free to act on your own. Become acquainted with your university's procedures.

4.4 Incompletes

The profession of teaching, while certainly a stimulating and rewarding one, is littered with nasty little details. One of these is the "incomplete". The theoretical purpose of an incomplete is to provide a vehicle for handling certain problem situations. Perhaps a student has completed a substantial amount of material in the course, but has been ill or has suffered a death in the family or some other setback. He needs to defer completion of the course work until the next term. The professor fills out an "Incomplete Form", and records the student's term grade as an "I" or "Incomplete", to formalize the understanding that the student will complete the work at some pre-specified future time. Many universities find it convenient to let professors administer incompletes as they see fit. As a result, there is much inconsistency and abuse.

Frankly, I've given a lot of incompletes in my life and very few of them were ever completed. Students get busy with the next semester's work, and never get around to things past. In fact I did not complete the only incomplete that I ever took as a student. It is also unfortunately the case that certain students will simply blow off a course and then ask for an incomplete at the end of the term. Often it is easier for you as the instructor to just grant the incomplete, given that an otherwise undisciplined student is not likely to complete it (the grade then usually, but not always, reverts to an "F"). You may very well wonder what is the point of engaging in a long interview with such a student to determine whether the incomplete is merited.

All this having been said, it is probably best, as with all matters in teaching that impinge on fairness, to have a uniform policy for handling incompletes. But think this through. Are you going to require that the student provide proof of his excuse? This sounds reasonable, but what if the student says, "My mother is dying of cancer." or "My grandmother just died and I cannot concentrate on my work." I know professors who will demand a letter from the physician or the undertaker, but this strikes me as a bit extreme. It could also prove to be uncomfortable or embarrassing for all concerned.

One convenient way to handle the request for an incomplete is to instruct the student to approach a professor teaching the same course the following term. The student should ask whether he can audit the course, having his work graded. The new professor of course will not submit an official grade for this student (after all, the student is not registered in his class). Instead, he will transmit the resulting grade to you. You then fill out a form to remove the student's Incomplete grade and replace it with that letter grade. This is clean and simple, and it works. You certainly don't want to have to re-teach some or all of the course for the benefit of just one student.

You are the academic analogue of a middle management executive in the business world. Executives exist, presumably, because they are smart enough to handle exceptional circumstances. Teaching is loaded with all of the sorts
of exceptions that are connected with dealing with people. I have used the
"incomplete" here as but one example of the problems and potential enigmas
that can arise. Your department probably has set policies, or at least guidelines,
for handling incompletes. Become acquainted with the routine procedures before
you give your first "I".

4.5 Frustration

One of the most commonly heard complaints of college mathematics instructors,
especially experienced instructors, is this: "Math 297 is a prerequisite for the
course that I am teaching yet the students don't seem to know anything from
Math 297." A variant of this is "My calculus students cannot\(^1\) add fractions" or
"My calculus students don't know how to expand \((a + b)^2\)."

Indeed, these are valid complaints. It is also valid to complain about the
high cost of living, or about death and taxes. The peccadillos described in the
first paragraph are facts of life and we as math instructors must deal with them.
The truth is that we instructors think about math all day long, every day. We
see the entire curriculum as a piece. For the experienced math instructor, there
are no seams and creases between linear algebra, calculus, differential equations,
and so forth. We swim effortlessly through the ideas, using whatever tools are
needed. (By the way, if this doesn't exactly describe you then don't panic—I'm
using a bit of poetic license here.) Students are different. They think about
math when they are in the math classroom and (one hopes) for a few designated
hours outside the classroom, but they are not inured in the subject.

So what is the point? It is simple. If you are in the second week of freshman
calculus and you need to add two algebraic expressions that are fractions, then
gently remind the students how to do it. If you need to expand the expression
\((a + b)^2\), then say, "You remember how this works—right?" After a few gentle
reminders, most of the students fall into the flow and they will remember how
it goes.

Take a break and watch the "Tonight Show" or "Late Night": Listen to
the monologue. If the host is going to crack a joke about someone slightly less
famous than Bill Clinton, then he gently reminds the audience who it is that he
is talking about. It’s just good sense. These television hosts can be even less
sure of how well informed their audiences are than we can be in our math classes.
They guarantee that their viewers will understand by providing a bridge.

Contrast these recommendations with the following rather common alterna-
tive. The professor needs to add two fractional algebraic expressions, so he just
barrels through it (rapidly and without comment, as one would do for a col-
league). After a few moments some hands are raised, some hesitant questions are
asked, and it soon becomes clear that many students are lost. The professor
says, "What is the matter with you people? This is high school stuff. Am I a

\(^1\)The Bank of America in Westwood Village, Los Angeles, used to regularly place an ad
vertisement in the UCLA student paper each spring. The purpose of the ad was to encourage
members of the latest graduating class to consider a career with the Bank of A. The ad read in part
"applicants must be able to add fractions." So it is not just math teachers who are plagued
by this problem.