

Syllabus MA831

Prof. Dirk Kreimer

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MA831 covers advanced topics in mathematical physics. It is an application of the material covered in MA557 to modern aspects of mathematical physics research. The course should be of equal interest to theoretical physics students with a background in particle physics or condensed matter, and to mathematicians with an interest in algebra, geometry or number-theory.

Prerequisites: MA557 or consent of instructor.

Material: D. Kreimer, *Dyson Schwinger equations: From Hopf algebras to number theory*, <http://arxiv.org/abs/hep-th/0609004>.

K. Ebrahimi-Fard and D. Kreimer, *Hopf algebra approach to Feynman diagram calculations*, J. Phys. A **38**, R385 (2005), <http://arxiv.org/abs/hep-th/0510202>.

More will be made available in class as necessary. Also, there will be a course homepage with links to literature as needed.

Grading: There will be two major homeworks and regular reading assignments. Class attendance (25%), following the reading assignments (25%) and doing the homework (50%) is mandatory for successful participation.

Homework: Two major homeworks will be handed out due after two weeks. You are welcome to work with others on your homework but all written work must be prepared independently.

Cheating: Boston University's policies on cheating and plagiarism are spelled out in the CAS Academic conduct code, available at CAS 105 and at <http://www.bu.edu/cas/academics/programs/conductcode.html>, and will be followed in this class.

Syllabus:

Week 1: Review of the basic concepts of field theory and the renormaliza-

tion group as used in physics.

Week 2: Review of these notions from an algebra perspective, using infinite dimensional Hopf- and Lie algebra.

Week 3: Scaling in complex graded Lie algebras.

Week 4: The diffeomorphism group and physical observables.

Week 5: The Ward identities of quantum electrodynamics as an Hopf ideal.

Week 6: The Slavnov–Taylor identities of quantum chromodynamics as an Hopf ideal.

Week 7: Application of the above to a concrete problem in quantum field theory as an example for the interplay between mathematics and physics.

Week 8: Recursive structure of non-perturbative Green functions and quantum equations of motion.

Week 9: Weierstrass preparation and the universal law for recursive systems.

Week 10: Extension to anomalous dimensions and determination of the Borel transform for renormalizable quantum field theories. Comparison with methods of constructive field theory.

Week 11: The appearance of non-rational numbers in such recursions. Comparison with the Drinfel'd associator.

Week 12: Application of the above to a concrete problem in non-perturbative quantum field theory. Comparison with other non-perturbative methods.

Week 13: Conformal invariance at a fix-point. The general structure of Mellin transforms. Conformal invariance and self-similarity in field theory and Schramm Loewner Evolution.

Week 14: Number-theoretic aspects of field theory. Relations between quantum field theory, algebraic and noncommutative geometry.