Assignment 1

MA557

Thu, Sep 28 2006. To be returned two weeks later.

1) Warming up: compute the coproduct and antipode for all rooted trees up to four vertices.
2) We introduced two ways of writing the coproduct for $T = B_+(X)$:

$$\Delta(B_+(X)) = B_+(X) \otimes 1 + (\mathrm{id} \otimes B_+)\Delta(X)$$

and

$$\Delta(T) = T \otimes 1 + 1 \otimes T + \sum_C P^C(T) \otimes R^C(T),$$

where the sum is over admissible subsets of edges C as explained in class. Prove that the two notions agree.
3) Define the tree factorial $T!$ by the vertex weights

$$T! = \prod_{v \in T^{[0]}} w(v).$$

Prove:

$$\frac{|T|}{T!} = \sum_{T' \in \mathcal{F}} \frac{1}{T'!}.$$

4) For Feynman rules

$$\phi(B_+(X))(a;\rho) = \int_0^\infty \frac{y^{-\rho}}{y+a} \phi(X)(y;\rho) dy$$

prove:

$$\phi(T) = a^{-|T|\rho} \prod_{v \in T^{[0]}} B_{w(v)},$$

where $B_k = \Gamma(k\rho)\Gamma(1 - k\rho)$.