1) Warming up: compute the coproduct and antipode for all rooted trees up to four vertices.

2) We introduced two ways of writing the coproduct for $T = B_+(X)$:

\[
\Delta(B_+(X)) = B_+(X) \otimes 1 + (\text{id} \otimes B_+) \Delta(X)
\]

and

\[
\Delta(T) = T \otimes 1 + 1 \otimes T + \sum_{C} P^C(T) \otimes R^C(T),
\]

where the sum is over admissible subsets of edges $C$ as explained in class. Prove that the two notions agree.

3) Define the tree factorial $T!$ by the vertex weights

\[
T! = \prod_{v \in T[0]} w(v).
\]

Prove:

\[
\frac{|T|}{T!} = \sum_{T' \in \mathcal{F}} \frac{1}{T'!}.
\]

4) For Feynman rules

\[
\phi(B_+(X))(\alpha; \rho) = \int_{0}^{\infty} \frac{y^{-\rho}}{y + \alpha} \phi(X)(y; \rho) dy
\]

prove:

\[
\phi(T) = a^{-|T| \rho} \prod_{v \in T[0]} B_{w(v)},
\]

where $B_k = \Gamma(k \rho) \Gamma(1 - k \rho)$. 

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Assignment 1

MA557

Thu, Sep 28 2006. To be returned two weeks later.