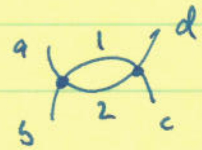
Assignment 2

1. For a 1PI graph Γ, define Feynman rules by

$$\phi(\Gamma) = \prod_{e \in \Gamma^{[1]}_{int}} \prod_{v \in \Gamma^{[0]}} \int \mathrm{Prop}(k_e)\, \delta^{(D)}\Big(\sum_{j \text{ adjacent } v} k_j\Big) \cdot d^D k_e$$

where the ~~stem~~ products are over internal edges and vertices and the sum is over all edges j, internal or external, adjacent to vertex v.

We count them as in-coming to v for each v, with an arbitrarily chosen orientation of edges.

Example $\Gamma =$ 

$$\phi(\Gamma) = \int \mathrm{Prop}(k_1)\, \mathrm{Prop}(k_2)\, \delta^{(D)}(k_a + k_b + k_1 + k_2) \cdot \delta^{(D)}(k_c + k_d - k_1 - k_2)\, d^D k_1\, d^D k_2 .$$

Prove: $$\phi(\Gamma) = \delta^{(D)}\Big(\sum_{j \in \Gamma^{[1]}_{ext}} k_j\Big) \int d^4 k_1 \dots d^4 k_r \cdot \mathrm{Int}(\Gamma).$$

where r is the first Betti number (number of independent cycles) and

$\mathrm{Int}(\Gamma)$ is a function of the external k_j, $j \in \Gamma^{(1)}_{ext}$, and r independent k_i, $i \in \Gamma^{(1)}_{int}$.

2) Let us give weights ω for edges and vertices for various theories as follows:

ϕ^4_4: $\omega(\text{—}) = 2$ $\omega(\text{+}) = 0$.

QED: $\omega(\sim\sim) = 2$, $\omega(\rightarrow) = 1$, $\omega(\text{photon–fermion vertex}) = 0$.

QCD: $\omega(\sim\sim) = 2$, $\omega(\text{-}\rightarrow\text{-}) = 2$, $\omega(\rightarrow) = 1$,

$\omega(\text{vertex}) = 0$, $\omega(\text{vertex}) = -1$, $\omega(\text{vertex}) = -1$,

$\omega(\text{vertex}) = 0$.

For edges and vertices in each theory as indicated, ~~constant~~ prove that these theories are renormalizable for $D = 4$.

(Prove $\omega(\Gamma) \equiv b_1 \cdot D - \sum$ weights is constant for any graph Γ with the same external edges).

3) $$\Delta \Gamma = \Gamma \otimes \mathbb{I} + \mathbb{I} \otimes \Gamma + \sum_{\gamma \subset \Gamma} \gamma \otimes \Gamma / \gamma .$$

With QCD weights given as above, and $\sum_{\gamma \subset \Gamma}$ a sum over all 1PI propo (disjoint unions of) subgraphs γ (with $\omega(\gamma) \geq 0$),

what is

$\Delta \Big($  $\Big) = \, ?$