SUMMER SCHOOL "THE STRUCTURE OF LOCAL QUANTUM FIELDS"

ORGS.: DIRK KREIMER WITH SPENCER BLOCH AND FRANCIS BROWN

Overview

In recent years we have seen many new insights into the mathematical structure of renormalizable quantum field theories. Such theories still form the core of theoretical physics underwritten by their ability to predict the outcome of physics experiments.

Progress was made in particular through understanding the mathematical structure of renormalization and the renormalization group [1]. The identification of a Hopf algebra structure in renormalization by Kreimer in 1997 [2], much elaborated on by Connes and Kreimer in the years 1998-2001, now leads to an understanding of the computational practice of perturbative quantum field theory in terms of algebraic geometry and mixed Hodge structures, starting with the work of Bloch-Esnault-Kreimer [3] and Bloch-Kreimer [4].

As a result, Francis Brown was able to streamline the computational techniques in Feynman diagram computations and to explain the connection to mixed Tate motives in a very impressive work recently [5, 6, 7], see also [8] by Aluffi and Marcolli, which directly connects to the detailed analysis of multiple zeta values and polylogarithms necessary in QFT computations [9, 10].

At the same time, from string theory we obtained the AdS/CFT correpondence which led to many new insights on the interplay between quantum fields and string theory. In particular, the Britto-Cachazo-Feng-Witten recursion relations [11] led to spectacular progress in conformally invariant N=4 super Yang–Mills theory, and many conjectures concerning the transcendental nature of Green functions in such theories were obtained by Beisert-Eden and Staudacher [12], by Korchemsky [13] and many others.

A connection between these two fields emerged recently through the understanding of the Hopf algebra structure of quantum gravity [14], and through the identification of co-ideals in the Hopf algebra structure of Green functions in a collaboration between van Suijlekom and Kreimer [15].

Furthermore, out of this grew an approach to insights into non-perturbative aspects of field theory [16, 17, 18] with its own connections to mathematics emerging via Ringel-Hall algebras and Stokes phenomena [19].

In view of these developments, we, Spencer Bloch, Francis Brown and Dirk Kreimer are organizing this three-week summer school in Les Houches.. At this moment, speakers who have agreed to lecture include

(1) **Spencer Bloch** (Chicago U.),
(2) **Ruth Britto** (CEA Saclay),
(3) **David Broadhurst** (Open U.),
(4) **Francis Brown** (Jussieu),
(5) **Nigel Glover** (Durham U., GB),
(6) **Gregory Korchemsky** (CEA Saclay),
(7) **Dirk Kreimer** (IHES),
(8) **Matthias Staudacher** (MPI Gravitation Potsdam),
(9) **Matt Szczesny, Boston U.**,
(10) **Walter van Suijlekom** (Nijmegen U.),
(11) **Karen Yeats** (Simon Fraser U.),

and we plan to invite a few more in the near future.

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