②

$$X = \prod_i X_i$$

$$\Delta X = \prod_i \Delta X_i$$

$$\Delta X_i = X_i \otimes \mathbb{I} + \mathbb{I} \otimes X_i + \sum_{C_i} P^{C_i}(X_i) \otimes R^{C_i}(X_i)$$

$$\Delta X = \prod_i (X_i \otimes \mathbb{I} + \mathbb{I} \otimes X_i)$$

$$+ \text{Rest}$$

$$\text{Rest} = \sum_{\hat{C}} P^{\hat{C}}(X) \otimes R^{\hat{C}}(X)$$

where $\hat{C}$ ~~contains a nonempt~~ is a non-empty subset of $\bigcup_i X_i^{[1]}$ such that $C_i = \hat{C} \cap X_i^{[1]}$ is admissible for each X_i.

$$P^{\hat{C}}(X) \otimes R^{\hat{C}}(X)$$

$$= \prod_{C_i \neq \emptyset} \left\{ P^{C_i}(X_i) \otimes R^{C_i}(X_i) \right\} \prod_{C_i = \emptyset} \left[X_i \otimes \mathbb{I} + \mathbb{I} \otimes X_i \right].$$

$$\Delta B_+ X = B_+ X \otimes \mathbb{I} + (\mathrm{id} \otimes B_+) \left\{ \prod_i X_i \otimes \mathbb{I} + \mathbb{I} \otimes X_i \right.$$

$$\left. + \text{Rest} \right\}. \Rightarrow \text{induction} \ \square$$

④ $X = \prod_i X_i$

$\phi(B_+ X)(a; \rho)$

$$= \int_0^\infty \frac{y^{-\rho}}{y+a} \, y^{-\rho \sum_i |X_i|} \prod_i \prod_{v_i \in X_i^{[0]}} B_{\omega(v_i)}$$

$$= a^{-\rho |T|} \prod_i \prod_{v_i \in X_i^{[0]}} B_{\omega(v_i)} \int_0^\infty \frac{y^{-\rho |T|}}{y+1}$$

$$= a^{-\rho |T|} \prod_{v \in T^{[0]}} B_{\omega(v)} \qquad \square$$

③ read section 5 of

ATMP 3, 627-670, 1999

(on course homepage)