

MA 123 - Exam 1 - Sample Questions from a Previous Exam - Prof. Meuser

1) Use the formula

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

to find $f'(x)$ for

$$f(x) = \frac{1}{x-1}$$

Show all work.

2) Consider the function $f(x) = 1 - |x|$.

a) Which of the following properties holds ?

i) $\lim_{x \rightarrow 0} f(x) = f(0)$ ii) $\lim_{x \rightarrow 0} f(x)$ exist but $\neq f(0)$ iii) $\lim_{x \rightarrow 0} f(x)$ does not exist.

Explain your answer.

b) Sketch the graph of $f'(x)$. Clearly indicate which points are included or excluded.

3) Find the following limits using properties of limits. Give reasons for your answer in each case.

a)

$$\lim_{x \rightarrow 0} \frac{(2+x)^2 - 4}{x}$$

b)

$$\lim_{x \rightarrow \pi/2} \frac{\sec x \cos x}{x}$$

c) If $\lim_{x \rightarrow a} f(x) = 3$ and $\lim_{x \rightarrow a} g(x) = 2$ find

$$\lim_{x \rightarrow a} \frac{f(x)^2 - 2g(x)}{g(x) - f(x)}$$

4) Let

$$f(x) = \begin{cases} x+1 & x \leq 0 \\ 1-x^2 & 0 < x \leq 2 \\ -x+1 & x > 2 \end{cases}$$

Find the following one-sided limits.

a) $\lim_{x \rightarrow 0^-} f(x)$

b) $\lim_{x \rightarrow 0^+} f(x)$

c) $\lim_{x \rightarrow 2^-} f(x)$

d) $\lim_{x \rightarrow 2^+} f(x)$

e) At what points is $f(x)$ not continuous ? On what intervals is $f(x)$ continuous?

5) A particle moves along a line with its position $s(t)$ at time t given by $s(t) = \sqrt{t}$.

a) What is its average velocity between $t = 0$ and $t = 2$?

b) Use a formula with limits to find its instantaneous velocity at $t = 1$.

6) Find the following limits using properties of limits. Give reasons for your answer in each case.

a)

$$\lim_{x \rightarrow 2} \frac{1 - x}{(x - 2)^2}$$

b)

$$\lim_{x \rightarrow \infty} \frac{x + 3}{\sqrt{9x^2 + 1}}$$