

1) (16 points)

a) Determine whether the **sequence**

$$a_n = \frac{n+1}{3n-1}$$

converges or diverges. If it converges, find the limit. Explain your answer.

b) Determine whether the **series** :

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n+1}{3n-1}$$

converges or diverges. If it converges, find the sum. Explain your answer.

2) (16 points)

a) Determine whether the **sequence**

$$a_n = \frac{3^{n+1}}{\pi^n}$$

converges or diverges. If it converges, find the limit. Explain your answer.

b) Determine whether the **series**

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{3^{n+1}}{\pi^n}$$

converges or diverges. If it converges, find the sum. Explain your answer.

3) (16 points) Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms and $\sum b_n$ is known to be divergent.

- If $a_n > b_n$ for all n can you conclude anything about the convergence or divergence of $\sum a_n$? Why or why not?
- If $a_n < b_n$ for all n can you conclude anything about the convergence or divergence of $\sum a_n$? Why or why not?

4) (16 points) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n}{e^n}$$

converges or diverges.

5) (16 points) Consider the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$$

- Does this series converge absolutely ? Why or why not?
- Does this series converge, but not absolutely? Why or why not?
- Does this series diverge ? Why or why not?

Label your answers for a), b) and c) separately and explain in each case.

QUESTION 6 IS ON THE BACK OF THIS PAGE

6) (16 points) Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$$

- a) Show this series converges absolutely.
- b) You wish to approximate the sum S of this series by S_n for some n . Find the **smallest** value of n that will give you an approximate value that is correct to three decimal places, i.e. the **smallest** value of n with

$$|R_n| = |S - S_n| \leq .0001$$

- c) Write S_n for the n you found in b) as an explicit sum of fractions (i.e. write the expression you would enter into your calculator if you wanted the decimal value). You should **NOT** give a decimal answer here and you do **NOT** have to compute a decimal answer .
- d) Is the approximation S_n that you wrote in c) larger or smaller than the actual value S of the series? Explain.