MA 124 - Fall '08 - Exam 3 - Prof. Meuser

1) (16 points) Find the radius **and** the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n n!}\, x^n$$

Explain any tests you are using to show convergence or divergence, including at the end-points of the interval.

2) (20 points)

a) Write the indefinite integral

$$\int \frac{x - \sin x}{x^3}\, dx$$

as a power series.
NOTE: Your answer must include a general formula for the coefficients.

b) Use your answer in a) to express the definite integral

$$\int_0^2 \frac{x - \sin x}{x^3}\, dx$$

as an infinite series.
Note: Your answer should make clear what the general term of the infinite series is.

c) Does the infinite series you wrote in b) converge or diverge? Explain.

3) (20 points)

a) Find the Maclaurin series of

$$f(x) = x \tan^{-1} x$$

b) Use your answer in a) to find the value of

$$f^{(8)}(0)$$

Your answer should be written as an explicit integer.

4) (20 points)

a) Use the binomial series to find the Maclaurin series of

$$f(x) = \frac{1}{\sqrt[3]{8 - x}}$$

You do **not** have to simplify the binomial coefficients.

b) What is the radius of convergence of this series? Explain.

c) What is the coefficient of x^2 in this series. Give an explicit value by computing the appropriate binomial coefficient and writing your answer as an explicit fraction a/b for a, b integers.

QUESTION 5 IS ON THE BACK OF THIS PAGE

5) (20 points)

a) Find the Taylor polynomial $T_4(x)$ of degree 4 for $f(x) = \cos x^2$ at $a = 0$.

Hint: You do **not** have to compute the derivatives if you know how to use a formula you have been given.

b) Use Taylor's inequality to estimate the accuracy of

$$\cos x^2 \approx T_4(x)$$

when $0 \leq x \leq \pi/4$.

You can use

$$f^{(5)}(x) = 160x^3\cos(x^2) + 120x\sin(x^2) - 32x^5\sin(x^2)$$

and your calculator.

b1) You first need a value of M with $|f^{(5)}(x)| \leq M$ for $0 \leq x \leq \pi/4$. Find an **integer** value of M that works and give reasons for your choice of M.

b2) Now use your choice of M to get an explicit **upper bound** for $|R_4(x)|$ which should first be written as a fraction. After writing it as a fraction you can give the **upper bound** as a decimal with **two** (i.e. not more than two) non-zero decimal digits. Show all work to get your answers.

c) If I use $T_4(x)$ to approximate $\cos x^2$ for $0 \leq x \leq \pi/4$ can I be sure that the error would be less than .25? Explain why or why not.

IMPORTANT NOTE

You should check your work to all of the above problems **CAREFULLY**. In particular you should be sure that you have followed the instructions and your answers are in the form requested.