MA 124 - Spring '09 - Exam 2 - Prof. Meuser

1) (16 points) Determine whether the improper integral

$$\int_{-\infty}^{\infty} \frac{x}{1+x^4}\, dx$$

converges or diverges. Write any necessary limits needed to determine your answer, and explain why those limits exist or do not exist using properties of limits.

2) (16 points) What can you say about the convergence of divergence of the infinite series $\sum a_n$ in each of the following cases and why?

a) $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \frac{1}{2}$
b) $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 1$
c) $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 2$

3) (16 points) The sum of the first 50 terms, S_{50}, is used to estimate the sum, S, of the convergent series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

a) Estimate the error $|R_{50}| = |S - S_{50}|$ using the Alternating Series Estimation Theorem.
b) Is $S_{50} > S$ or $S_{50} < S$? Explain.

4) (16 points) Determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

converges or diverges. Explain your answer and make clear which test you are using.

5) (16 points)

a) Determine whether the **sequence**

$$a_n = \frac{n}{2n+1}$$

converges or diverges. If it converges, find the limit. Explain your answer.

b) Determine whether the **series**

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n}{2n+1}$$

converges or diverges. Explain your answer.

QUESTION 6 IS ON THE BACK OF THIS PAGE

6) (16 points)

a) Determine whether the **sequence**

$$a_n = (-1)^n \frac{\sqrt{n}}{1+n}$$

converges or diverges. If it converges, find the limit. Explain your answer.

b) Determine whether the **series**

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{1+n}$$

converges absolutely, converges but not absolutely or diverges. Explain your answer and make clear any tests you are applying.