MA 124 - Spring '09 - Exam 3 - Prof. Meuser

1) (16 points) Find the radius **and** the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \sqrt{n}\; x^n$$

Explain any tests you are using to show convergence or divergence, including at the endpoints of the interval.

2) (16 points) Find the Maclaurin series for

$$f(x) = \frac{x}{4 + x^2}$$

and its radius of convergence.

NOTE: Your answer must be written in the form $\sum_{n=0}^{\infty} c_n x^n$ and include a general formula for all the coefficients c_n.

3) (16 points)

a) If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ for all x, what is the formula for c_n ? Why?

b) Let $f(x) = \sin(x^2)$. We know:

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!}\; x^{4n+2}$$

Use this and your answer in a) to find $f^{(10)}(0)$.

TIP: Be careful which coefficients you are using!

4) (16 points) Use the binomial series to find the Maclaurin series of

$$f(x) = \frac{x^2}{(1-x)^3}$$

a) First give an answer **without** simplifying the binomial coefficients.

b) Next simplify the binomial coefficients to find a general formula for **all** of the coefficients.

c) Use your answer in b) to find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(n+2)(n+1)}{2^{n+3}}$$

QUESTIONS 5 AND 6 ARE ON THE BACK OF THIS PAGE

5) (16 points)

a) Find the Taylor polynomial $T_3(x)$ of degree 3 for $f(x) = xe^{-2x}$ at $a = 0$.

Hint: You do **not** have to compute the derivatives if you know how to use a formula you have been given.

b) Use Taylor's inequality to estimate the accuracy of

$$xe^{-2x} \approx T_3(x)$$

when $|x| \leq 0.1$.

You can use

$$f^{(4)}(x) = e^{-2x}(16x - 32)$$

and your calculator.

b1) Find a value of M with $|f^{(4)}(x)| \leq M$ for $|x| \leq 0.1$. Give reasons why your choice of M is correct.

b2) Now use your choice of M to get an explicit **upper bound** for $|R_3(x)|$ which should first be written as a fraction. After writing it as a fraction you can give the **upper bound** as a decimal with **two** (i.e. not more than two) non-zero decimal digits. Show all work to get your answers.

6) (16 points)

a) Put $z = \sqrt{3} + i$ and $w = 1 + \sqrt{3}i$ in polar form $r(\cos\theta + i\sin\theta)$ where you give the values of θ corresponding to z and w.

Then find the polar forms of zw and z/w where you give their values of θ.

b) Use DeMoivre's Theorem:

$$z^n = (r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$$

to find the value of $w^6 = (1 + \sqrt{3}i)^6$ in both polar form and $a + bi$ form.

c) Find the cube roots of i in both polar form and $a + bi$ form.

c) Give the value of

$$e^{i\frac{\pi}{3}}$$

in the form $a + bi$ where any values of $\cos\theta$ and $\sin\theta$ are given.

IMPORTANT NOTE

You should check your work to all of the above problems **CAREFULLY**. In particular you should be sure that you have followed the instructions and your answers are in the form requested.