

MA 129 - Homework - October 4, 2007

Sequences

Applying Limit Laws to Sequences

Problems from Stewart: 8.1: 11, 16, 18, 20, 22, 28

1) Determine whether or not the following sequences  $a_n$  converge or diverge.

a)  $\frac{n!}{n^n}$                       b)  $\frac{2^n + (-1)^n}{2^{n+1} + (-1)^{n+1}}$

c)  $\frac{(-1)^n \sqrt{n} \sin(n^n)}{n+1}$

d)  $\frac{2^n}{n!}$                       e)  $\frac{2^{n^2}}{n!}$

Bounded Monotonic Sequences

Problems from Stewart: 8.1: 41 - 44, 47

Problems from Stewart (with extra questions asked):

- 38) Do a), c) as stated. In addition:  
b) Just use the definition. You don't have to estimate the value.  
d) (EXTRA CREDIT): Show the sequence has a (finite) limit.

40) Note that the sequence can be defined by:

$$a_1 = \sqrt{2} \qquad a_{n+1} = \sqrt{2a_n}$$

Answer:

- a) Show the sequence has a (finite) limit. Hint: Use induction and show

$$\text{If } 0 < a_n < 2 \text{ then } a_n < \sqrt{2a_n} < 2$$

- 50) Do a) and b) as stated. In addition do:  
c) Show that with  $f(x) = \cos x$  and  $a_1 = 1$  the sequence defined by

$$a_{n+1} = \cos(a_n)$$

has a limit.

Additional Note on 50: The function  $f(x) = \cos x$  satisfies  $f(x)$  differentiable on  $[0, 1]$ ,  $0 \leq f(x) \leq 1$  on  $[0, 1]$ . Thus this function is a specific example of one you had on your previous homework problems (and the exam!), where you showed there was an  $x$  in  $[0, 1]$  with  $f(x) = x$ . Your argument above gives another proof of this.

However you should also note that this previous result doesn't help to show c) above.

(In fact there is **exactly one**  $x$  in  $[0, 1]$  with  $\cos(x) = x$ , using  $f'(x) \neq 1$  on  $[0, 1]$ , which was also on your homework (and the exam!).)