MA 129 - Homework - September 13, 2007

Differentiable Functions

Problems from Stewart: 2.7: 35, 36 (Use the definition of $f'(0)$ as a limit).
2.8: 47
Chapter 2 Review Exercises: 30
Chapter 3 Review Exercises: 75

1) Find $f'(0)$ if

$$f(x) = \begin{cases} g(x)\sin(\frac{1}{x}) & x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and $g(0) = g'(0) = 0$.

NOTE: Stewart 2.7/ 36 is a specific example.

2) Suppose $f(x) = x \cdot g(x)$ for some function $g(x)$ which is continuous at 0. Prove $f(x)$ is differentiable at 0 and find $f'(0)$ in terms of g.

3) Can 2) be applied to Stewart 2.7/36 or not? I.e. is there a function $g(x)$ in this case so that 2) can be applied? If so, what is $g(x)$? If not, why not?

4) Suppose $f(x)$ is differentiable at 0 and $f(0) = 0$. Prove $f(x) = x \cdot g(x)$ for some function $g(x)$ which is continuous at 0.

Hint: Try $g(x) = f(x)/x$.