

MA 129 - Homework - September 25, 2007

Fundamental Theorem of Calculus

Problems from Stewart (assigned on September 20):

5.2: 17, 19, 50, 51

5.4: 11, 15, 17, 25

Chap. 5 Review: 8, 39, 69, 70

Additional Problems on the Fundamental Theorem of Calculus
(assigned on September 25):

1) If $f(x)$ is continuous on $[a, b]$, let

$$F(x) = \int_a^x f(t) dt \quad a \leq x \leq b$$

If c is in (a, b) are the following true or false? If true explain why, if false give a counterexample.

- a) If f is differentiable at c then F is differentiable at c .
- b) If f is differentiable at c then F' is continuous at c .
- c) If f' is continuous at c then F' is continuous at c .

2) If f is continuous, g and h are differentiable and

$$F(x) = \int_{g(x)}^{h(x)} f(t) dt$$

then find a formula for $F'(x)$. Hint:

$$\int_{g(x)}^{h(x)} f(t) dt = \int_{g(x)}^0 f(t) dt + \int_0^{h(x)} f(t) dt$$

. Use the Chain Rule !

Improper Integrals
Assigned on September 25

Problems from Stewart: 5.10: 41, 43, 45, 46 (Hint for 45, 45: Let $u = 1/x$), 47 - 49.

1) Determine whether the following integrals converge or diverge:

a) $\int_0^{\infty} \frac{1}{1+x^2} dx$ b) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

c) $\int_0^{\infty} \frac{1}{\sqrt{1+x^3}} dx$ d) $\int_0^{\infty} \frac{x}{1+x^{3/2}} dx$

e) $\int_0^{\infty} \frac{1}{x\sqrt{1+x}} dx$

2) Show

$$\int_0^{\infty} x^r dx$$

never converges.

NOTE: Write

$$\int_0^{\infty} x^r dx = \int_0^1 x^r dx + \int_1^{\infty} x^r dx$$

and consider the cases

- a) $r \geq 0$
- b) $-1 < r < 0$
- c) $r = -1$
- d) $r < -1$