

Let ' s use Mathematica to find some prime numbers between  $10^{10}$  and  $10^{16}$ .

First we note that by the Prime Number Theorem there are approximately  $x / \ln x$  prime numbers less than or equal to  $x$ . Note that  $\ln 10^k = k \ln 10$ , and the command for natural logarithm is `Log[ ]`. `N[ ]` gives an actual numerical value. We first find the approximate number of primes  $< 10^{16}$ , and then the approximate number of primes  $< 10^{10}$ , and subtract them to find the approximate number of primes between  $10^{10}$  and  $10^{16}$

```
In[27]:= N[10^16 / (16 Log[10]) - 10^10 / (10 Log[10])]
```

```
Out[27]= 2.71434 × 1014
```

The above is the approximate number of primes between  $10^{10}$  and  $10^{16}$ . So there are a LOT of primes in this range. If we choose an ODD number at random in this range, since there are  $(10^{16} - 10^{10}) / 2$  odd numbers in this range, we can calculate the approximate probability that an odd number in this range will be prime to be :

```
In[28]:= 2 * (N[10^16 / (16 Log[10]) - 10^10 / (10 Log[10])]) / (10^16 - 10^10)
```

```
Out[28]= 0.0542868
```

So if we choose an odd number at random in this range the chances of it being a prime are only slightly more than 5%, i.e. on average we might have to choose 20 odd numbers at random before we found a prime. Let ' s try one to see if we are lucky by choosing a random integer between  $10^{10}$  and  $10^{16}$  (if it is even we can either try again or divide it by 2 and see if it is odd and still in range).

```
In[31]:= Random[Integer, {10^10, 10^16}]
```

```
Out[31]= 2 055 259 494 792 511
```

We can ask Mathematica whether or not and integer is prime using the `PrimeQ` command.

```
In[32]:= PrimeQ[2 055 259 494 792 511]
```

```
Out[32]= False
```

If we are not lucky by choosing an odd number at random, in fact we can ask Mathematica directly for a random prime in a specified range using the `RandomPrime` command.

```
In[33]:= RandomPrime[{10^10, 10^16}]
```

```
Out[33]= 8 914 109 010 080 107
```

Just to be sure it is prime we apply the `PrimeQ` command.

```
In[34]:= PrimeQ[8 914 109 010 080 107]
```

```
Out[34]= True
```

We choose another one :

```
In[35]:= RandomPrime[{10^10, 10^16}]
```

```
Out[35]= 7 769 020 344 007 919
```

```
In[36]:= PrimeQ[7 769 020 344 007 919]
```

```
Out[36]= True
```

**Now let 's label our two primes p and q  
(so we can just enter these letters when doing further calculations) .**

```
In[37]:= p = 8 914 109 010 080 107
```

```
Out[37]= 8 914 109 010 080 107
```

```
In[38]:= q = 7 769 020 344 007 919
```

```
Out[38]= 7 769 020 344 007 919
```

**Finally we compute our product m = pq**

```
In[39]:= m = p * q
```

```
Out[39]= 69 253 894 248 016 643 181 947 632 367 333
```

**We label this value m. This will be OUR m (for class directory listing)**

**Note that most hand calculators do not store this many digits,  
so even if your hand calculator has enough digits for p and q,  
it may not have enough digits to return the value of their product exactly.**