1) (16 points) Let \( p \) be a prime number. Show the only solutions of \[ x^3 \equiv x \pmod{p} \]
are \( x \equiv 0, \pm 1 \pmod{p} \).

2) (16 points) An astronomer knows that a satellite orbits the earth in a period that is an exact multiple of 1 hour that is less than 1 day. If the astronomer notes that the satellite completes 11 orbits in an interval that starts when a 24-hour clock reads 0 hours and ends when the clock reads 17 hours, how long is the orbital period of the satellite?

3) (16 points)
   a) Find the smallest positive integer \( n \) that satisfies:
      \[ 2 \mid n, \ 3 \mid (n + 1), \ 4 \mid (n + 2), \ 5 \mid (n + 3), \ 6 \mid (n + 4) \]
      Make clear how you are using congruences to find your answer.
   b) What is the next largest integer that satisfies a) and why?

4) (16 points) Find the highest power of 5 that divides the following integers. Also determine whether they are divisible by 3 and by 9 Explain your answer to both question using divisibility tests involving the integers in their decimal representation.
   a) 11259375
   b) 1145573945375

5) (16 points)
   a) What is the value of \( 11^{16} + 17^{10} \pmod{11 \cdot 17} \) ?
   b) If \( p \) and \( q \) are distinct primes, what do you think the value of \( p^{q-1} + q^{p-1} \pmod{p \cdot q} \) is equal to? (Note: a) is an example).
   c) \textbf{Prove} your answer in b). Your answer should make clear \textbf{what theorems} you are applying and \textbf{where} you are applying

6) (16 points) Show every composite Fermat number \[ F_n = 2^{2^n} + 1 \]
is a pseudoprime to the base 2.
   Hint: Note that \( 2^{2^n} \equiv -1 \pmod{F_n} \).

NOTE: In the computational problems, especially 2), you should check your work carefully since partial credit will be based on correct work only up to your \textbf{first} error.