In[35]:=

```
w1 = {-4, 3 - 2 I, I, 1 - 4 I}
```

Out[35]= $\{-4, 3-2i, i, 1-4i\}$

In[36]:=

```
w2 = {-1 - 5 I, 5 - 4 I, -3 + 5 I, 7 - 2 I}
```

Out[36]= $\{-1-5i, 5-4i, -3+5i, 7-2i\}$

In[37]:=

```
w3 = {-27 - I, -7 - 6 I, -15 + 25 I, -7 - 6 I}
```

Out[37]= $\{-27-i, -7-6i, -15+25i, -7-6i\}$

We begin the Gram - Schmidt process to transform {w1, w2, w3} into an orthogonal set {v1, v2, v3}. Note that the inner product of v and w , is given by v.cw where we take the conjugate of the entries in w.

In[38]:=

```
v1 = w1
```

Out[38]= $\{-4, 3-2i, i, 1-4i\}$

In[41]:=

```
cv1 = Conjugate[v1]
```

Out[41]= $\{-4, 3+2i, -i, 1+4i\}$

In[42]:=

```
v2 = w2 - (w2.cv1 / v1.cv1) v1
```

Out[42]= $\{3-i, -5i, -2+4i, 2+i\}$

In[43]:=

```
cv2 = Conjugate[v2]
```

Out[43]= $\{3+i, 5i, -2-4i, 2-i\}$

We check v1 is orthogonal to v2.

In[44]:=

```
v1.cv2
```

Out[44]= 0

In[45]:=

```
v3 = w3 - (w3.cv1 / v1.cv1) v1 - (w3.cv2 / v2.cv2) v2
```

Out[45]= $\{-17-i, -9+8i, -18+16i, -9+8i\}$

We check v3 is orthogonal to v1 and v2.

In[46]:=

```
v3.cv1
```

Out[46]= 0

In[47]:=

```
v3.cv2
```

Out[47]= 0

So {v1, v2, v3} is an orthogonal basis. We divide by the lengths to get an orthonormal basis.

In[48]:=
```
Sqrt[v1.cv1]
```
Out[48]= $\sqrt{47}$

In[49]:=
```
Sqrt[v2.cv2]
```
Out[49]= $2\sqrt{15}$

In[51]:=
```
cv3 = Conjugate[v3]
```
Out[51]= $\{-17+i, -9-8i, -18-16i, -9-8i\}$

In[52]:=
```
Sqrt[v3.cv3]
```
Out[52]= $2\sqrt{290}$

We let {b1, b2, b3} be the orthonormal basis.

In[54]:=
```
b1 = (1/Sqrt[47]) v1
```
Out[54]= $\left\{-\frac{4}{\sqrt{47}}, \frac{3-2i}{\sqrt{47}}, \frac{i}{\sqrt{47}}, \frac{1-4i}{\sqrt{47}}\right\}$

In[62]:=
```
b2 = (1/(2 Sqrt[15])) v2
```
Out[62]= $\left\{\frac{\frac{3}{2}-\frac{i}{2}}{\sqrt{15}}, -\frac{1}{2} i \sqrt{\frac{5}{3}}, -\frac{1-2i}{\sqrt{15}}, \frac{1+\frac{i}{2}}{\sqrt{15}}\right\}$

In[56]:=
```
b3 = (1/(2 Sqrt[290])) v3
```
Out[56]= $\left\{-\frac{\frac{17}{2}+\frac{i}{2}}{\sqrt{290}}, -\frac{\frac{9}{2}-4i}{\sqrt{290}}, -\frac{9-8i}{\sqrt{290}}, -\frac{\frac{9}{2}-4i}{\sqrt{290}}\right\}$

We check that {b1, b2, b3} is orthonormal. We already know the vectors are orthogonal, so we just need to check that they have length 1.

In[58]:=
```
cb1 = Conjugate[b1]
```
Out[58]= $\left\{-\frac{4}{\sqrt{47}}, \frac{3+2i}{\sqrt{47}}, -\frac{i}{\sqrt{47}}, \frac{1+4i}{\sqrt{47}}\right\}$

In[59]:=
```
b1.cb1
```
Out[59]= 1

In[63]:=
```
cb2 = Conjugate[b2]
```
Out[63]= $\left\{\frac{\frac{3}{2}+\frac{i}{2}}{\sqrt{15}}, \frac{1}{2} i \sqrt{\frac{5}{3}}, -\frac{1+2i}{\sqrt{15}}, \frac{1-\frac{i}{2}}{\sqrt{15}}\right\}$

In[64]:=

```
b2.cb2
```

Out[64]= 1

In[65]:=

```
cb3 = Conjugate[b3]
```

Out[65]= $\left\{-\frac{\frac{17}{2}-\frac{i}{2}}{\sqrt{290}}, -\frac{\frac{9}{2}+4\,i}{\sqrt{290}}, -\frac{9+8\,i}{\sqrt{290}}, -\frac{\frac{9}{2}+4\,i}{\sqrt{290}}\right\}$

In[66]:=

```
b3.cb3
```

Out[66]= 1

So {b1, b2, b3} is our orthonormal bases.

We now calculate the Fourier coefficients of x where

In[67]:=

```
x = {-13 - 7 I, -12 + 3 I, -39 - 11 I, -26 + 5 I}
```

Out[67]= $\{-13-7\,i, -12+3\,i, -39-11\,i, -26+5\,i\}$

In[68]:=

```
x.cb1
```

Out[68]= $(-1-i)\sqrt{47}$

In[69]:=

```
x.cb2
```

Out[69]= $(-2+4\,i)\sqrt{15}$

In[70]:=

```
x.cb3
```

Out[70]= $(2+2\,i)\sqrt{290}$

Let ' s check these

In[71]:=

```
(x.cb1) b1 + (x.cb2) b2 + (x.cb3) b3
```

Out[71]= $\{-13-7\,i, -12+3\,i, -39-11\,i, -26+5\,i\}$

Since this is x they are correct.