MATH 114 QUIZ 1
13 SEPTEMBER 2016

Solve the following two problems. Show all steps in your work.

(1) Simplify the following expression. Express your answer as a single fraction with no negative exponents.

$$\frac{3^3(r^{-2}s)^{-2}t^{-3}}{6^2s^{-1}r^5}$$

First, observe that $(r^{-2}s)^{-2} = (r^{-2})^{-2}s^{-2} = r^4s^{-2}$. Also, $c^{-n} = \frac{1}{c^n}$ for any nonzero real number c, so rewriting negative exponents in this way, we get

$$\frac{3^3(r^{-2}s)^{-2}t^{-3}}{6^2s^{-1}r^5} = \frac{3^3r^4s^{-2}t^{-3}}{6^2s^{-1}r^5} = \frac{3^3r^4\frac{1}{s^2}\frac{1}{t^3}}{6^2\frac{1}{s}r^5} = \frac{3^3r^4s}{6^2r^5s^2t^3} = \frac{3^3}{3^2\cdot 2^2rst^3} = \frac{3}{4rst^3}.$$

(2) Use polynomial long division to divide $y^3 - 4y^2 - 3$ by $-y + 5$. Check your work by verifying that $y^3 - 4y^2 - 3$ really is equal to

$$(\text{quotient})\cdot(-y+5) + (\text{remainder}).$$

We first carry out the long division algorithm:

$$\begin{array}{r|rrrr}
 & -y^2 & -y & -5 & \\ \hline
-y+5 & y^3 & -4y^2 & & -3 \\
 & -y^3 & +5y^2 & & \\ \hline
 & & y^2 & & \\
 & & -y^2 & +5y & \\ \hline
 & & & 5y & -3 \\
 & & & -5y & +25 \\ \hline
 & & & & 22
\end{array}$$

To check our answer, notice that

$$\begin{aligned}
(-y^2 - y - 5)\cdot(-y+5) + 22 &= -y^2(-y+5) - y(-y+5) - 5(-y+5) + 22 \\
&= (y^3 - 5y^2) + (y^2 - 5y) + (5y - 25) + 22 \\
&= y^3 - 4y^2 - 3,
\end{aligned}$$

which is what we want.