MATH 114 QUIZ 10 SOLUTIONS
6 DECEMBER 2016

Solve the following two problems. Justify all of your work with clearly written mathematics.

(1) Compute the exact value of $\tan(\cos^{-1}(4/5))$.

By definition, $\cos^{-1}(4/5)$ is the angle in the interval $[0, \pi]$ such that the corresponding point on the unit circle has x-coordinate $4/5$. Since $4/5 > 0$, this point is in the first quadrant, and has y-coordinate satisfying

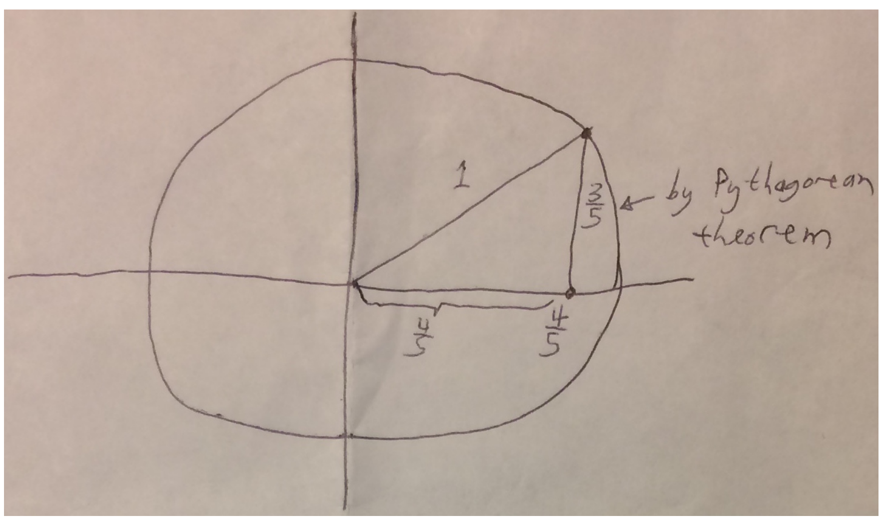
$$(4/5)^2 + y^2 = 1$$

by the Pythagorean theorem. So

$$y^2 = 1 - (4/5)^2 = 9/25 = (3/5)^2,$$

and since (x, y) is in the first quadrant, $y > 0$, so $y = 3/5$. Hence,

$$\tan(\cos^{-1}(4/5)) = \frac{y}{x} = \frac{3/5}{4/5} = \frac{3}{4}.$$



(2) Find all solutions to the equation $\sin(\theta)^2 = 5\sin(\theta)^2 - 3$ in the interval $0 \leq \theta < 2\pi$.

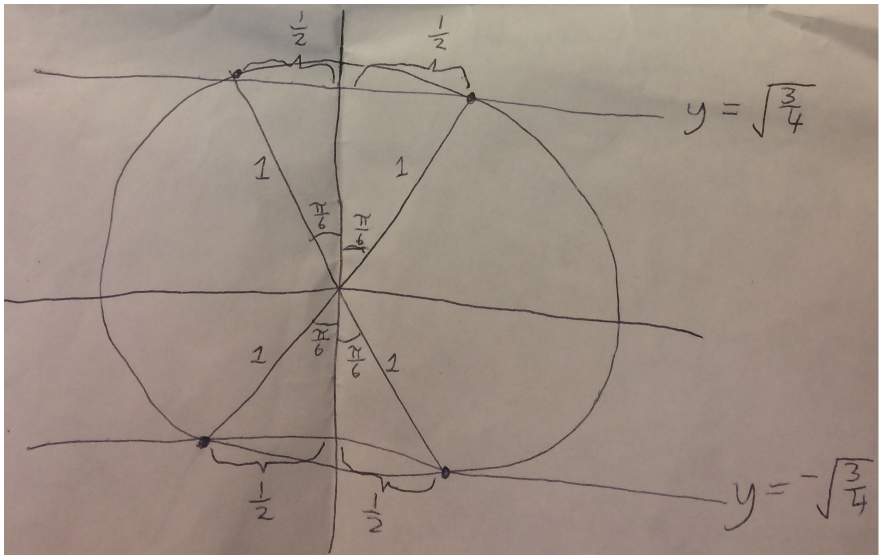
The given equation is equivalent to

$$4\sin(\theta)^2 = 3,$$

so $\sin(\theta)^2 = 3/4$, meaning

$$\sin(\theta) = \pm\sqrt{3/4}.$$

Let $(x, y) = (\cos(\theta), \sin(\theta))$ be the point on the unit circle corresponding to θ. Then $y = \sin(\theta) = \pm\sqrt{3/4}$, and by the Pythagorean theorem, $x = \pm 1/2$. Thus, the points $(0, 0)$, (x, y), and $(-x, y)$ form an equilateral triangle, so (x, y) makes an angle of $\pi/6$ with either the positive or negative y-axis (depending on whether y is positive or negative). So there are four possibilities for θ: $\frac{\pi}{2} - \frac{\pi}{6}$, $\frac{\pi}{2} + \frac{\pi}{6}$, $\frac{3\pi}{2} - \frac{\pi}{6}$, and $\frac{3\pi}{2} + \frac{\pi}{6}$.


$\frac{1}{2}$
$\frac{1}{2}$
$y = \sqrt{\frac{3}{4}}$
1
1
$\frac{\pi}{6}$
$\frac{\pi}{6}$
1
$\frac{\pi}{6}$
$\frac{\pi}{6}$
1
$\frac{1}{2}$
$\frac{1}{2}$
$y = -\sqrt{\frac{3}{4}}$