# MATH 114 QUIZ 2
# 20 SEPTEMBER 2016

**Solve the following two problems. Show all steps in your work.**

(1) Solve the following inequality for $x$. Write your answer using interval notation.

$$4 \leq |3x - 5|$$

If $3x - 5 \geq 0$, then $|3x - 5| = 3x - 5$, so we have

$$4 \leq 3x - 5,$$

which implies $9 \leq 3x$, so $3 \leq x$.

If $3x - 5 < 0$, then $|3x - 5| = -(3x - 5)$, so we have

$$4 \leq -(3x - 5),$$

which implies $3x - 5 \leq -4$, so $3x \leq 1$, so $x \leq \frac{1}{3}$.

Thus, $x \geq 3$ or $x \leq \frac{1}{3}$. In interval notation, the set of all $x$ making the original inequality true is

$$(-\infty, \tfrac{1}{3}] \cup [3, +\infty).$$

(2) Express the following as a single fraction (with no fractions in the numerator or denominator). Check that your answer has the same value as the below expression when a specific number (of your choice) is substituted for $x$. (This can catch many (but not all) errors.)

$$\frac{5 - \dfrac{3}{x+1}}{x + \dfrac{x+2}{x-1}}$$

First observe that

$$5 - \frac{3}{x+1} = \frac{5(x+1)}{x+1} - \frac{3}{x+1} = \frac{5(x+1) - 3}{x+1} = \frac{5x + 5 - 3}{x+1} = \frac{5x+2}{x+1}$$

and

$$x + \frac{x+2}{x-1} = \frac{x(x-1)}{x-1} + \frac{x+2}{x-1} = \frac{x(x-1) + x + 2}{x-1}$$
$$= \frac{x^2 - x + x + 2}{x-1} = \frac{x^2+2}{x-1}.$$

So

$$\frac{5-\dfrac{3}{x+1}}{x+\dfrac{x+2}{x-1}} = \frac{\left(\dfrac{5x+2}{x+1}\right)}{\left(\dfrac{x^2+2}{x-1}\right)} = \frac{\left(\dfrac{5x+2}{x+1}\right)\cdot(x+1)}{\left(\dfrac{x^2+2}{x-1}\right)\cdot(x+1)} = \frac{5x+2}{\left(\dfrac{x^2+2}{x-1}\right)\cdot(x+1)}$$
$$= \frac{(5x+2)\cdot(x-1)}{\left(\dfrac{x^2+2}{x-1}\right)\cdot(x+1)\cdot(x-1)} = \frac{(5x+2)\cdot(x-1)}{(x^2+2)\cdot(x+1)}.$$

Now let's check, using $x = 0$ (because it's easy):

$$\frac{5-\dfrac{3}{0+1}}{0+\dfrac{0+2}{0-1}} = \frac{5-3}{0-2} = \frac{2}{-2} = -1,$$

and

$$\frac{(5\cdot 0+2)\cdot(0-1)}{(0^2+2)\cdot(0+1)} = \frac{2\cdot(-1)}{2\cdot 1} = \frac{-2}{2} = -1.$$

Since these are the same, this gives us some evidence (though not absolute proof) that we didn't make an error earlier on.

To gain more confidence in our answer, we could test more numbers — it still doesn't prove with certainty that the answer is correct, but you can catch a lot of errors this way.