MATH 114 QUIZ 3 SOLUTIONS
27 SEPTEMBER 2016

Solve the following two problems. Show all steps in your work.

(1) Find an equation for the line passing through the point $(3, 2)$ and parallel to the line given by $5 = 4y - 2x$.

If $5 = 4y - 2x$, then $2x + 5 = 4y$ (by adding $2x$ to both sides), so $\frac{2}{4}x + \frac{5}{4} = y$ (by dividing both sides by 4). So our line has slope $\frac{2}{4} = \frac{1}{2}$, and therefore must be given by an equation of the form

$$y = \frac{1}{2}x + c$$

for some number c. Also, our line must pass through the point $(3, 2)$; plugging $x = 3$ and $y = 2$ into the above equation, we get

$$2 = \frac{1}{2}(3) + c,$$

so $c = 2 - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2}$. So the line passing through the point $(3, 2)$ and parallel to the line $5 = 4y - 2x$ is given by the equation

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}.}$$

(2) For the following equation, list all the x-intercepts and y-intercepts, and test for each of the three kinds of symmetry discussed in class.

$$18 = 9x + 2y^2$$

- The *x-intercepts* are the points where the line intersects the x-axis. The x-axis is the line $y = 0$. If $y = 0$, then
$$18 = 9x + 2(0)^2 = 9x,$$
so $2 = x$. So there is exactly one x-intercept, the point $(2, 0)$.
- The *y-intercepts* are the points where the line intersects the y-axis. The y-axis is the line $x = 0$. If $x = 0$, then
$$18 = 9(0) + 2y^2 = 2y^2,$$
so $9 = y^2$, which means y is either 3 or -3. So there are exactly two y-intercepts, the point $(0, 3)$ and the point $(0, -3)$.
- An equation has *symmetry across the x-axis* if replacing y with $-y$ gives an equivalent equation. Replacing y with $-y$, we get
$$18 = 9x + 2(-y)^2.$$
Since $(-y)^2 = y^2$, this is equivalent to the original equation.
- An equation has *symmetry across the y-axis* if replacing x with $-x$ gives an equivalent equation. Replacing x with $-x$, we get
$$18 = 9(-x) + 2y^2 = -9x + 2y^2.$$

This is *not* equivalent to the original equation. Indeed, $x = 2$ and $y = 0$ is a solution to the original equation, but not to $18 = -9x + 2y^2$, so they can't be equivalent. (Equivalent equations have *exactly* the same set of solutions.)

- An equation has *symmetry across the origin* if replacing x with $-x$ *and* y with $-y$ gives an equivalent equation. Replacing x with $-x$ and y with $-y$, we get

$$18 = 9(-x) + 2(-y)^2 = -9x + 2y^2.$$

Exactly as above, this is not equivalent to the original equation.

In summary, the equation $18 = 9x + 2y^2$ has x-intercept $(2, 0)$, has y-intercepts $(0, 3)$ and $(0, -3)$, has x-axis symmetry, does not have y-axis symmetry, and does not have origin symmetry.