

MATH 114 QUIZ 4 SOLUTIONS
11 OCTOBER 2016

Solve the following two problems. Show all steps in your work.

- (1) Consider the quadratic function g given by

$$g(x) = 6x - 5 - 2x^2$$

for all real numbers x . Complete the square to find the axis of symmetry and the vertex of the function:

$$\begin{aligned} g(x) &= -2x^2 + 6x - 5 = -2(x^2 - 3x) - 5 \\ &= (-2) \left(x^2 - 3x + \left(\frac{3}{2}\right)^2 \right) - (-2) \left(\frac{3}{2}\right)^2 - 5 \\ &= -2 \left(x - \frac{3}{2} \right)^2 + 2 \left(\frac{3}{2}\right)^2 - 5 \\ &= -2 \left(x - \frac{3}{2} \right)^2 + \frac{9}{2} - 5 = -2 \left(x - \frac{3}{2} \right)^2 - \frac{1}{2}. \end{aligned}$$

This tells us the axis of symmetry is $x = 3/2$ and the vertex is $(3/2, -1/2)$.

Since the leading coefficient, -2 , is negative, the parabola $y = g(x)$ faces downward, so g has no minimum. The maximum of g occurs when $x = 3/2$, and the maximum value is $-1/2$.

- (2) Suppose h is a linear function with x -intercept $(3/7, 0)$ and y -intercept $(0, 3)$. Give a sequence of transformations (shifting, compressing, stretching, and/or reflections) that transforms the line $y = x$ into the line $y = h(x)$.

The slope of the line $y = h(x)$ is

$$\frac{3 - 0}{0 - \frac{3}{7}} = \frac{3}{-\frac{3}{7}} = 3 \cdot \frac{7}{-3} = -7.$$

The y intercept is given, so

$$h(x) = -7x + 3$$

for all real numbers x . We give the following sequence of transformations:

- Start with $y = x$.
- Stretch by a factor of 7 vertically: $y = 7x$.
- Reflect across the x -axis: $y = -7x$.
- Shift up by 3 units: $y = -7x + 3 = h(x)$.