MATH 114 QUIZ 5 SOLUTIONS
18 OCTOBER 2016

Solve the following two problems. Show all steps in your work.

(1) Solve the following inequality. Write your answer in interval notation.

$$2x^2 - 5 < x + 10$$

Subtracting $x + 10$ from both sides, the above inequality is equivalent to

$$2x^2 - x - 15 < 0.$$

To find where $2x^2 - x - 15$ can transition from positive to negative, we first find where

$$2x^2 - x - 15 = 0.$$

To do this, we factor $2x^2 - x - 15$. Using the ac method, we want to factor $2(-15) = -30$ as a product of two numbers whose difference is -1; we can use -6 and 5. Keeping this in mind, observe that

$$2x^2 - x - 15 = 2x^2 - 6x + 5x - 15 = 2x(x - 3) + 5(x - 3) = (2x + 5)(x - 3).$$

So $2x^2 - x - 15 = 0$ if and only if $2x + 5 = 0$ or $x - 3 = 0$, which is to say, exactly when $x = -\frac{5}{2}$ or $x = 3$.

This partitions the real line into three regions: $(-\infty, -\frac{5}{2})$, $(-\frac{5}{2}, 3)$, and $(3, \infty)$. To solve the original inequality, we must determine whether $2x^2 - x - 15$ is positive or negative on each of these regions. We do so by trying a number in each region, namely -3, 0, and 4:

- $2(-3)^2 - (-3) - 15 = 2 \cdot 9 + 3 - 15 = 6 > 0$
- $2(0)^2 - 0 - 15 = -15 < 0$
- $2(4)^2 - 4 - 15 = 2 \cdot 16 - 4 - 15 = 13 > 0$

So $2x^2 - x - 15 < 0$ exactly when x is in the interval $(-\frac{5}{2}, 3)$. (The endpoints are not included because we don't want the cases where $2x^2 - x - 15$ is *equal* to zero.)

(2) Consider a rectangle with one vertex at the origin, one vertex on the positive x-axis, one vertex on the positive y-axis, and one vertex on the line $2x + y = 6$. What is the *largest* area that such a rectangle could possibly have?

Write (a, b) for the upper right vertex of the rectangle. Then the area of this rectangle is ab. We know that $2a + b = 6$, so $b = 6 - 2a$, so the area is

$$ab = a(6 - 2a) = 6a - 2a^2.$$

The answer we want is the largest possible value of $6a - 2a^2$, which occurs at the vertex of the parabola

$$y = 6x - 2x^2.$$

Completing the square, we have

$$\begin{aligned} 6x - 2x^2 &= -2(x^2 - 3x) = -2(x^2 - 3x + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2) \\ &= -2(x^2 - 3x + (\tfrac{3}{2})^2) + 2(\tfrac{3}{2})^2 \\ &= -2(x - \tfrac{3}{2})^2 + 2 \cdot \tfrac{9}{4} = -2(x - \tfrac{3}{2})^2 + \tfrac{9}{2}. \end{aligned}$$

So the vertex occurs at $(\frac{3}{2}, \frac{9}{2})$. This tells us the largest possible area is $\frac{9}{2}$.