MATH 114 QUIZ 6
25 OCTOBER 2016

Solve the following two problems. Show all steps in your work.

(1) Find all vertical, horizontal, and oblique asymptotes, if any, for the following function:

$$F(x) = \frac{(3x+6)(2x^2+7x-4)}{(x^2+6x+8)(x-1)}$$

Notice that

$$x^2+6x+8 = (x+4)(x+2)$$

and

$$2x^2+7x-4 = (2x-1)(x+4).$$

(You can discover this by trial and error, the ac method, the rational root theorem, completing the square, or the quadratic formula.) So for all x in the domain of F,

$$F(x) = \frac{(3x+6)(2x-1)(x+4)}{(x+4)(x+2)(x-1)} = \frac{(3x+6)(2x-1)}{(x+2)(x-1)}.$$

Also, $3x+6 = 3(x+2)$, so

$$F(x) = \frac{3(x+2)(2x-1)}{(x+2)(x-1)} = \frac{3(2x-1)}{(x-1)}.$$

So F has only one vertical asymptote, at $x=1$. (Note that $2x-1$ is not zero when $x=1$, so there's no more cancellation.) Also,

$$F(x) = \frac{3(2x-1)}{(x-1)} = \frac{(6x-3)}{(x-1)} = \frac{(6x-3)\frac{1}{x}}{(x-1)\frac{1}{x}} = \frac{6-\frac{3}{x}}{1-\frac{1}{x}},$$

which approaches 6 as $x \to \infty$ or $x \to -\infty$. Thus, F has a horizontal asymptote at $y=6$, and no oblique asymptotes.

(2) Solve the following inequality. Check your answer by testing the inequality for at least two specific values of x. Write your answer in interval notation.

$$3 \leq \frac{2x-5}{x+4}$$

The above inequality is equivalent to

$$0 \leq \frac{2x-5}{x+4} - 3,$$

so we must first find where $\frac{2x-5}{x+4} - 3$ has zeroes or vertical asymptotes. Observe that

$$\frac{2x-5}{x+4} - 3 = \frac{2x-5}{x+4} - \frac{3(x+4)}{x+4} = \frac{2x-5-3(x+4)}{x+4} = \frac{-x-17}{x+4}.$$

Thus, there is a vertical asymptote when $x+4=0$, and a zero when $-x-17=0$. So we must test a point in each of the intervals $(-\infty,-17)$, $(-17,-4)$, and $(-4,\infty)$:

- If $x = -20$, then

$$\frac{-x-17}{x+4} = \frac{-(-20)-17}{-20+4} = \frac{20-17}{-20+4} = \frac{3}{-16} < 0.$$

- If $x = -10$, then
$$\frac{-x-17}{x+4} = \frac{-(-10)-17}{-10+4} = \frac{10-17}{-10+4} = \frac{-7}{-6} = \frac{7}{6} > 0.$$
- If $x = 0$, then
$$\frac{-x-17}{x+4} = \frac{-0-17}{0+4} = \frac{-17}{4} < 0.$$

Therefore,
$$\frac{2x-5}{x+4} - 3 = \frac{-x-17}{x+4} > 0$$
exactly when x is in the interval $(-17, -4)$. Including the zero at $x = -17$,
$$\frac{2x-5}{x+4} - 3 = \frac{-x-17}{x+4} \geq 0$$
exactly when x is in the interval $[-17, -4)$. So
$$3 \leq \frac{2x-5}{x+4}$$
if and only if x is in the interval $[-17, -4)$.