MATH 114 QUIZ 7 SOLUTIONS
8 NOVEMBER 2016

Solve the following two problems. Show all steps in your work.

(1) Which real numbers x satisfy $\frac{1}{9^x} = 3^{4x-5}$? Check your answers.

Suppose x is a real number such that

$$\frac{1}{9^x} = 3^{4x-5}.$$

Since $9 = 3^2$, we have $9^x = (3^2)^x = 3^{2x}$, so

$$3^{-2x} = \frac{1}{3^{2x}} = \frac{1}{9^x} = 3^{4x-5}.$$

Exponentiation is one-to-one (the base 3 logarithm is the inverse function), so this implies

$$-2x = 4x - 5.$$

Adding $2x + 5$ to both sides gives us $5 = 6x$, so $x = \frac{5}{6}$.

Checking our answer,

$$4 \cdot \frac{5}{6} - 5 = \frac{20}{6} - 5 = \frac{20}{6} - \frac{30}{6} = \frac{-10}{6},$$

so $3^{4 \cdot \frac{5}{6}} = 3^{-10/6}$, while

$$\frac{1}{9^{5/6}} = 9^{-5/6} = (3^2)^{-5/6} = 3^{-10/6},$$

so $x = \frac{5}{6}$ is indeed a solution.

(2) Find the range and any horizontal asymptotes of the function $F(x) = 10 - e^{2x-6}$. Also, what's $F(3)$?

As $x \to -\infty$, we have $2x - 6 \to -\infty$, so $e^{2x-6} \to 0$, so $F(x) \to 10$, giving a horizontal asymptote at $y = 10$. However, $e^{2x-6} > 0$ for all x, so $F(x) < 10$ for all x.

As $x \to \infty$, we have $2x - 6 \to \infty$, so $e^{2x-6} \to \infty$, so $F(x) \to -\infty$ (notice that e^{2x-6} is *subtracted* in the formula for F).

Putting these together, $F(x)$ can be any real number less than 10: the range is $(-\infty, 10)$.

Finally, $F(3) = 10 - e^{2\cdot 3-6} = 10 - e^0 = 10 - 1 = 9.$