MATH 114 QUIZ 8 SOLUTIONS
15 NOVEMBER 2016

Solve the following two problems. Show all steps in your work.

(1) Find all real numbers r such that the following equation is true. Check your answers.

$$\log_5(r) - \log_5(7) = \log_5(r+3) - 2\log_5(7)$$

Suppose r is a solution to the above equation. Using properties of logarithms, we have:

$$\log_5(r) - \log_5(7) = \log_5(r/7)$$

and

$$\log_5(r+3) - 2\log_5(7) = \log_5(r+3) - \log_5(7^2) = \log_5\left(\frac{r+3}{7^2}\right),$$

so

$$\log_5\left(\frac{r}{7}\right) = \log_5\left(\frac{r+3}{7^2}\right).$$

The base 5 logarithm has an inverse function, the base 5 exponential, so $\log_5$ is one-to-one. Therefore,

$$\frac{r}{7} = \frac{r+3}{7^2}.$$

Solving this, we obtain $7r = r+3$, so $6r = 3$, so $r = 1/2$. And, indeed,

$$\begin{aligned}\log_5(1/2+3) - 2\log_5(7) &= \log_5(7/2) - 2\log_5(7)\\ &= \log_5(7) - \log_5(2) - 2\log_5(7) = -\log_5(2) - \log_5(7),\end{aligned}$$

while

$$\log_5(1/2) - \log_5(7) = \log_5(2^{-1}) - \log_5(7) = -\log_5(2) - \log_5(7),$$

so $r = 1/2$ really is a solution (and the only solution).

(2) Find the domain, range, and any asymptotes of the following function:

$$g(x) = 5 - \log_2(4-x)$$

For x to be in the domain of g, we must have $4 - x > 0$, so the domain is $(-\infty, 4)$.

The range can be computed in two ways.[1] One way is to observe that the graph $y = g(x)$ can be obtained from $y = \log_2(x)$ by transformations. Here's one such sequence of transformations:

(a) Shift left 4 units: $y = \log_2(4+x)$
(b) Reflect across y-axis: $y = \log_2(4-x)$
(c) Reflect across x-axis: $y = -\log_2(4-x)$
(d) Shift up 5 units: $y = 5 - \log_5(4-x) = g(x)$

[1]Probably more than two ways, actually, but two ways that you've learned in this class.

The range of $\log_2$ is $\mathbb{R}$, and neither shifts nor reflections change this, so the range of g is still $\mathbb{R}$.

Another approach is to compute the inverse function g^{-1}, whose graph is

$$x = g(y) = 5 - \log_2(4 - y).$$

Solving for y, we obtain:

$$\begin{aligned} x &= 5 - \log_2(4 - y) \\ \log_2(4 - y) &= 5 - x \\ 4 - y &= 2^{5-x} \\ 4 - 2^{5-x} &= y \end{aligned}$$

So $g^{-1}(x) = 4 - 2^{5-x}$, which has domain $\mathbb{R}$. The domain of the inverse function is the range of the original function, so we again find the range of g is $\mathbb{R}$.

As for asymptotes, we can use the above transformations: $\log_2$ has a vertical asymptote at $x = 0$ and no other asymptotes. Shifting left moves the asymptote to $x = -4$, reflecting across the y-axis moves the asymptote to $x = 4$, and the reflection across the x-axis and the upward shift don't affect the asymptote. So, g has a vertical asymptote at $x = 4$ and no other asymptotes.