MATH 114 QUIZ 9 SOLUTIONS
22 NOVEMBER 2016

Solve the following two problems. Show all steps in your work. You may use the Pythagorean theorem without proof.

(1) Compute the exact values of $\cos(7\pi/4)$ and $\sin(7\pi/4)$.

By definition, $(\cos(7\pi/4), \sin(7\pi/4))$ is the point on the unit circle at angle $7\pi/4$. This corresponds to an isosceles right triangle with hypotenuse 1 in the fourth quadrant.

By the Pythagorean theorem, the side lengths of an isosceles right triangle with hypotenuse 1 are both $1/\sqrt{2}$, since $c^2 + c^2 = 1$ implies $c^2 = 1/2$, so $c = 1/\sqrt{2}$.

Since the point is in the fourth quadrant, the x-coordinate is positive and the y-coordinate is negative. Hence,

$$\cos(7\pi/4) = \frac{1}{\sqrt{2}}$$

and

$$\sin(7\pi/4) = \frac{-1}{\sqrt{2}}.$$

(2) Compute the exact value of $3\cos(2\pi/17)\cos(2\pi/17) + 3\sin(2\pi/17)\sin(2\pi/17)$.

Observe that

$$\begin{aligned}
&3\cos(2\pi/17)\cos(2\pi/17) + 3\sin(2\pi/17)\sin(2\pi/17) \\
&= 3\cos(2\pi/17)^2 + 3\sin(2\pi/17)^2 \\
&= 3\left(\cos(2\pi/17)^2 + \sin(2\pi/17)^2\right).
\end{aligned}$$

By the Pythagorean theorem and the definition of cosine and sine,

$$\cos(t)^2 + \sin(t)^2 = 1$$

for *every* real number t; in particular, setting $t = 2\pi/17$,

$$\cos(2\pi/17)^2 + \sin(2\pi/17)^2 = 1,$$

so

$$3\left(\cos(2\pi/17)^2 + \sin(2\pi/17)^2\right) = 3 \cdot 1 = 3.$$